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Study of unsteady velocity field of MHD Couette Flow

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Abstract

In this paper we have considered the oscillatory flow performing small harmonic oscillation in a channel of gap h. Our analysis is based on Ishigaki[5,6] work in which, he has assumed the stream unsteady flow with its conjugate complex part. We obtained the amplitude of the fluctuating velocity gradients its variation with respect to transfer magnetic field.

An exact analysis of oscillatory MHD Couette flow has been performed by using series solution method. Numerical calculations are made for phase angle and shearing stress of the unsteady MHD flow in a horizontal channel for various values of Hartman number.

Key words : Couette Flow, Phase angle, Shearing Stress, Hartman number.

Introduction

Oscillatory free convective flow problem in the presence of a magnetic field through a porous medium have attracted the attention of a number of scholars because of their possible application in many branches of sciences and technology. In practice, cooling of porous structure is achieved by forcing the liquid or gas through capillaries of solid. Actually, they are used to insulate a heated body to maintain its temperature. Study of origin of flow through a porous medium is heavily based on Darcy's experimental law. The oscillatory flow past a porous bed was studied by Chawala and Singh[2]. Raptis *et al.*[9] presented the study two dimensional flow of viscous fluid through a porous medium bounded by a porous surface subjected to a constant suction velocity by taking account of free convection currents. Borzini *et al.*[1] and Han[4] have studied hydromagnetic free convection flow of radiating fluids.

Vajravelu[15] studied this aspect and obtained exact solution of MHD flow in horizontal channel. He extended this type of work for the general solution. Following Lal[7] an attempt has been made to obtain exact solution of a MHD flow in a horizontal channel by adopting method initiated by Lighthill[8] for oscillatory flow.

One of the exact solutions of the Navier-Stokes equation in which no restriction is imposed on the amplitude and frequency was obtained by Stuart[13] for the flow past a flat plate with free stream oscillation.

Laminar boundary layer in unsteady magneto-hydrodynamics flows has been studied by Girshick and Kruger[3], Tokis[14], Soundalgaker and Uplekar[11] and Watanke[16]. Their works are confined to the problem of boundary layer with free stream oscillation. The magnetic analogue of Lighthills analysis was done by Soundalgekar[12]. We have considered the oscillatory flow performing small harmonic oscillation in a channel of gap h. Our analysis is based on Ishigaki[5 and 6] work in which, he has assumed the stream unsteady flow with its conjugate complex part. We obtained the amplitude of the fluctuating velocity gradients its variation with respect to transfer magnetic field.

An exact analysis of oscillatory MHD Couette flow has been performed by using series solution method. Numerical calculations are made for phase angle and shearing stress of the unsteady MHD flow in a horizontal channel for various values of Hartman number.

Mathematical Analysis :

Consider two-dimensional hydromagnetic unsteady oscillatory flow of incompressible viscous fluid of small electrical conductivity through a horizontal channel bounded by two infinite parallel flat plates of which one is at rest and the other is oscillating in its own plane with a constant mean velocity.

Take x-axis along the stationary plate and y-axis normal to it. Assume that the flow is independent of the distance along the wall and the velocity component normal to the wall is zero.

According to Shercliff[10], the induced magnetic field is neglected which is justified for small magnetic Reynolds number flow.

The Navier–Stokes equation of motion in a co-ordinate system fixed with the wall oscillating with the velocity U(t) gives

$$\frac{\partial u}{\partial t}=\frac{\partial U}{\partial t}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2(u-U)}{\rho} \qquad (1.1)$$

where u the velocity component along x-axis, ρ the density, ν the kinematic viscosity, U(t) the velocity of the oscillating wall in the x-direction, B_0 the imposed uniform magnetic field, t the time, y the normal distance.

We consider the case in which unsteady velocity is given by

$$U(t)=U_h \operatorname{Re}\sum_{r=0}^{n} e^{i\omega rt}\in^r \qquad (1.2)$$

Where ω is the frequency and U_h and $\in$ are constant, Re means real.

We then look for a solution of the form

$$u=U_h \operatorname{Re}\sum_{r=0}^{n}\in^r G(\eta)\, e^{i\omega rt} \qquad (1.3)$$

In which $\eta = \frac{y}{h}$

Substituting equations (1.2) and (1.3) in (1.1) and equating the coefficient of $\in^{r} e^{i\omega r t}$, we obtain

$$G''(\eta) - [M + i f r] G(\eta) = -[M + i f r] \qquad (1.4)$$

where $M = \frac{\sigma B_0^2}{\rho} \frac{h^2}{\nu}$ and $f = \frac{\omega h^2}{\nu}$ is magnetic and frequency parameter respectively and primes denotes differentiation w.r.t. η.

Now to find the solution of equation (1.4) with boundary conditions

η	=	0,	G(η)	= 0
η	=	1,	G(η)	= 1

The solution of equation (1.4) is

$$G''(\eta) - \ell^2 G(\eta) = -\ell^2$$

where $\ell^2 = M + i f r$

$$\ell^2 = M + i f r$$

P. I. = 1

then the solution

$$G(\eta) = C_1 e^{\ell \eta} + C_2 e^{-\ell \eta} + 1 \qquad (1.5)$$

If η = 0, G(η) = 0 then

$$C_1 + C_2 + 1 = 0 \qquad (1.6)$$

If η = 1, G(η) = 1 then

$$C_1 e^{\ell} + C_2 e^{-\ell} = 0 \qquad (1.7)$$

Solving equation (1.6) and (1.7), we get

$$C_1 = \frac{-e^{-\ell}}{e^{\ell} - e^{-\ell}}$$

$$C_2 = \frac{e^{\ell}}{e^{\ell} - e^{-\ell}}$$

From equation (1.5)

$$G(\eta) = 1 - \cosh \ell \eta + \coth \ell \sinh \ell \eta \qquad (1.8)$$

Where $\ell = \sqrt{(M + i f r)}$

The unsteady velocity

$$u = U_h \operatorname{Re} \sum_{r=0}^{n} \in^{r} [1 - \cosh \ell \eta + \coth \ell \sinh \ell \eta] e^{i\omega r t} \qquad (1.9)$$

The shear stress at y = 0 is

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$$\tau_w = \ell U_h \mu \operatorname{Re} \sum_{r=0}^{n} [\coth \ell . \cosh \ell \eta - \sinh \ell \eta]_{y=0} \, t^{i\omega r t} \in^{r} \qquad (1.10)$$

$$\bar{\tau}_w = \frac{T_\omega}{\frac{U_h \mu}{h}} = \operatorname{Re}[\ell \coth \ell] \in^{r} e^{i\omega r t} \qquad (1.11)$$

When ωt = 0

$$\bar{\tau}_w = \operatorname{Re}[\ell \coth \ell] \in^{r} \qquad (1.12)$$

When $\omega t = \frac{\pi}{2}$

$$\bar{\tau}_w = \operatorname{Re}[\ell \coth \ell] \in^{r} e^{i r \frac{\pi}{2}} \qquad (1.12')$$

The magnitude of the fluctuating velocity gradient at lower wall

$$|u'| = \frac{\cosh A \sin B [A \sinh A \cos B + B \cosh A \sin B]}{(\sinh^2 A \cos^2 B - \cosh^2 A \sin^2 B)} \qquad (1.13)$$

At upper wall

$$|u'| = \frac{\sin 2B \cosh 2A [A \sinh A \cos B + B \cosh A \sin B]}{(\sinh^2 A \cos^2 B - \cosh^2 A \sin^2 B)} \qquad (1.14)$$

Where A = $e^{X} \cos y$

B = $e^{X} \sin y$

$$X = \frac{1}{2} (M^2 + f^2 r^2)^{1/2}$$

$$Y = \frac{1}{2} \tan^{-1} \left(\frac{f r}{M} \right)$$

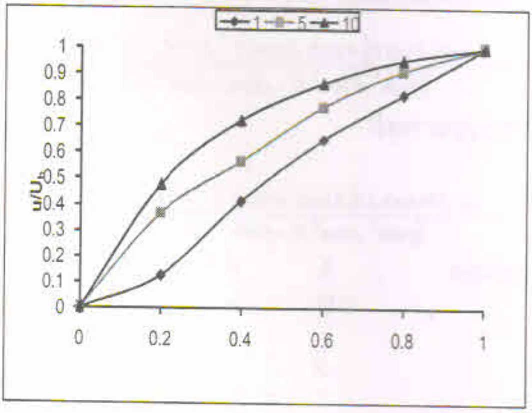
Results and Discussion

The variations of unsteady velocity u/Uh in the medium for different values of magnetic field parameter M have been shown in Tables and Graph. Cases for $\omega t=0$ and $\omega t=\pi/2$ are shown. Shear stress has been calculated from equations (1.12) and (1.12'). Values are entered in Table–1.4, 1.5 and 1.6. Corresponding variation for $\omega t=0$ and $\omega t=\pi/2$ have been shown with the help of Graphs.

Table-1.1 Unsteady velocity for different values of M

$\omega t = 0$, $\in = 0.01$, $f = 0.1$, $r = 0$

M / η	u/Uh		
	1	5	10
0.0	0.000	0.000	0.0000
0.2	0.126	0.3712	0.4786
0.4	0.415	0.5691	0.7232
0.6	0.654	0.7797	0.8653
0.8	0.823	0.9131	0.9536
1.0	1.000	1.000	1.0000

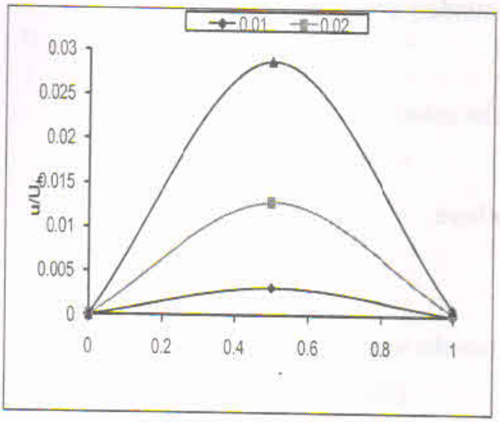


Graph- 1.1: Variation of unsteady velocity for different values of M with η.

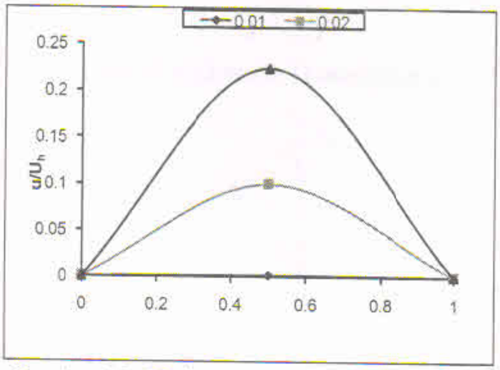
Table-1.2. Unsteady velocity with η for different values of ∈

$r = 2$, $\omega t = 0$, $f = 0.1$,

∈ / η	u/Uh		
	0.01	0.02	0.03
For M = 1			
0.0	0.0000	0.0000	0.0000
0.5	0.0023	0.0128	0.0288
1.0	0.0001	0.0004	0.0009
For M = 5			
0.0	0.0000	0.0000	0.0000
0.5	0.0024	0.0998	0.2246
1.0	0.0001	0.0004	0.0009



Graph- 1.2: Variation of unsteady velocity with η for different values of ∈.

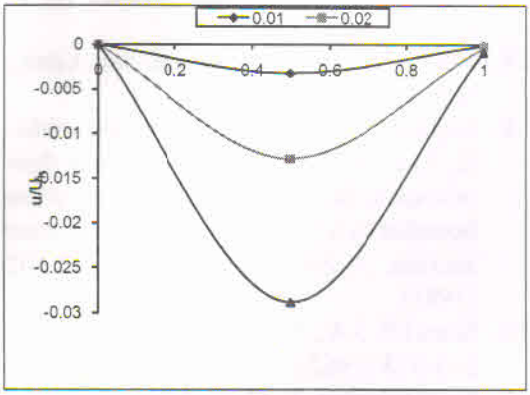


Graph- 1.3: Variation of unsteady velocity with η for different values of ∈.

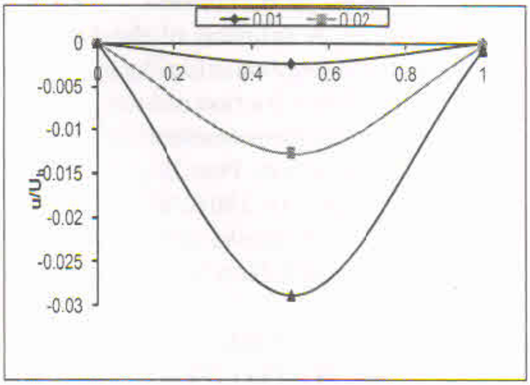
Table-1.3

$r = 2$, $\omega t = \pi/2$, $f = 0.1$,

$\in$ / η	u/Uh		
	0.01	0.02	0.03
For M = 1			
0.0	0.0000	0.0000	0.0000
0.5	-0.0032	-0.0128	-0.0288
1.0	-0.0001	-0.0004	-0.0009
For M = 5			
0.0	0.0000	0.0000	0.0000
0.5	-0.0024	-0.0128	-0.0288
1.0	-0.0001	0.0004	-0.0009



Graph- 1.4: Variation of unsteady velocity $\omega t = \pi/2$ for different values of $\in$.

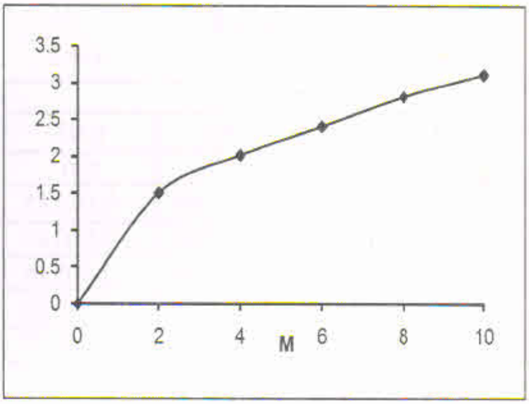


Graph- 1.5: Variation of unsteady velocity $\omega t = \pi/2$ for different values of $\in$.

Table-1.4

$f = 0.1$, $\omega t = 0$, $r = 0$, $\in = 0.01$

M	$\tilde{\tau}_{\omega}$
0	0.0
2	1.5
4	2.0
6	2.4
8	2.8
10	3.1

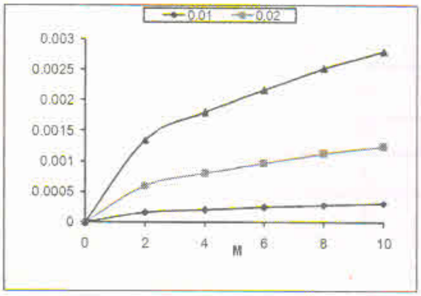


Graph- 1.6: Variation of shear stress M for $\omega t = 0$,

Table-1.5

$f = 0.1$, $\omega t = 0$, $r = 2$

$\in$ / M	$\tilde{\tau}_{\omega}$		
	0.01	0.02	0.03
0.0	0.0	0.0	0.0
2.0	0.00015	0.00060	0.00135
4.0	0.00020	0.00080	0.00180
6.0	0.00024	0.00096	0.00216
8.0	0.00028	0.00112	0.00252
10.0	0.00031	0.00124	0.00279

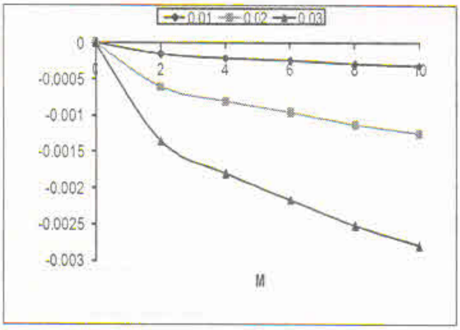


Graph-1.7: Variation of shear stress for $\omega t = 0$ with respect to M and $\in$.

Table-1.6

$f = 0.1$, $\omega t = \pi/2$, $r = 2$

$\in$ \ M	τ_w		
	0.01	0.02	0.03
0.0	0.0	0.0	0.0
2.0	−0.00015	−0.00060	−0.00135
4.0	−0.00020	−0.00080	−0.00180
6.0	−0.00024	−0.00096	−0.00216
8.0	−0.00028	−0.00112	−0.00252
10.0	−0.00031	−0.00124	−0.00279



Graph-1.8: Variation of shear stress for $\omega t = \pi/2$ with respect to M and $\in$.

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