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Formulation of a Graph Theoretic Model to Find Currents in Electrical Circuits

CHANCHAL BORUAH¹ and CHANDRA CHUTIA²

¹Assistant Professor

Department of Mathematics, Arya Vidyapeeth College, Guwahati-781016 (India)

²Associate Professor

Department of Mathematics, Jorhat Institute of Science and Technology, Jorhat-785010 (India)

Email of Corresponding Author: Chanchal.boruah@rediffmail.com

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Abstract

Mathematics plays vital role in various fields. Graph theory is a branch in Mathematics and its results have applications in many areas of the computing, social and natural sciences. Here we study about some objects (points) connected by a particular relation among themselves. With the help of graph theory many mathematical models have been formulated to solve real life problems in different fields. Different kinds of optimization can be done with the help of graph theory in Physics, Chemistry, Computer Science, Communication Science, Electrical Engineering etc. Electrical circuits can be studied and simplified by using many well known theorems of Electrical Engineering. But it seems to be laborious to use the existing and traditional methods to find branch currents and loop currents of electrical circuits if it contains many branches. For this kind of difficulties graph theory is useful and powerful technique. Different kinds of optimization can be done with the help of graph theory. In this paper, we have formulated a graph theoretic model to find the currents of an electrical circuits containing finite number of 'n' branches.

Keywords: Graph theory, Electrical circuits, Impedance matrix, Branch current, Loop current.

2010 Mathematics subject classification: 05C50

1. Introduction

In recent years, graph theory has been well-developed and becomes worthy of study by its own

right. Here we have focused on the application of graph theory in electrical circuit analysis. An electrical circuit is a collection of electrical components and devices. A graph representation of electrical circuit is represented

in terms of points called vertices or nodes and lines called edges or branches. We apply the concept of graph theory to solve electrical circuit problems. The behaviour of electrical circuits generally depends upon the characteristics of each of internally connected elements and their connection rules. The connection rule gives a relation between electrical circuits with graph theory. A connected graph without closed path i.e. tree was implemented by Kirchhoff in 1847. He used graph theoretical concept in the calculation of currents in network or circuits. In 1892 this concept was improved by J.C. Maxwell¹⁷. The relation between Kirchhoff's methods for constructing the equations that describe an electric circuit and graphs was first clarified by Veblen in 1916. In this paper, we have developed a graph theoretic model for calculation of currents of electrical circuits containing 'n' branches by using topological form of Kirchhoff's Voltage Law.

2. Review of Graph Theory in Electrical Circuit Analysis :

In recent years electronic simulation programs are used to determine the circuit equations by means of nodal analysis which shows a more direct way to describe the relation between graphs and electric circuits. In 1980, Satoru Fujishige developed an algorithm to solve graph realization problem. He used the fundamental circuit matrix or graphic matrix for developing it¹⁸. Istvan Fary in 1983 studied and developed some algorithms on generalised circuits¹⁶. In 1985, Karl Gustafson studied graph and networks using vector calculus and developed some results relating to divergence and curl²⁰. In 1997, Gunther and Hoscheck adapted ROW method for simulation of electric circuits¹⁹. In 2005 R.B. Bapat⁶ studied incidence matrix, Laplacian and distance matrix for a tree with attached graphs. They found some results for the minors of the incidence matrix and the Laplacian and for the inverse and the determinant of the distance matrix. In 2010, Giuseppe Ali proposed a model for integrated circuits using partial differential equation and algebra³. In 2010, Diestel¹³ gave a topological approach for studying locally finite graphs that crucially incorporates their ends. Chang¹¹ in 2011 presented a model based on matrix in power supply networks. A review of fundamental and historic results of algebraic graph theory and electrical network analysis is given in¹⁵.

3. From Circuit to Graph :

A graph can be obtained from a circuit. We identify the graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by $\{i, j\}$ ignoring the direction. Similarly the notation (i, j) can be used for oriented edges, where 'i' is the start vertex and 'j' is the end vertex. In general, current source and voltage source are replaced by open circuit and short circuit respectively. It may be observed that a branch may contain more than one element. The choice of branches and nodes with respect to network element is flexible.

Figure 1. A circuit and its corresponding graph

3.1 Matrices Associated to a Graph :

A graph can be represented using matrices. Following are some examples of matrices obtained from graph.

a) Fundamental Tie Set Matrix (Fundamental Loop Matrix) $[B]$ ¹⁰

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branches of tree). Here, all loops obtained from a particular tree form the rows and all branches form the columns such that

$$b_{ij} = \begin{cases} 1, & \text{if branch } b_j \text{ in loop } i \text{ is in the same direction as the loop} \\ -1, & \text{if branch } b_j \text{ in loop } i \text{ is in the opposite direction of the loop} \\ 0, & \text{if branch } b_j \text{ is not in loop } i \end{cases}$$

b) Branch Impedance Matrix $[Z_b]$ ¹⁰

It is a square matrix of order 'm' where m is the number of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entry is zero.

3.2 Application of Graph Theory in Network Equilibrium Equations¹⁰:

If V_{SK} be the voltage source in a branch k having impedance z_k and carrying current i_k , then the branch voltage $v_k = z_k i_k + V_{SK}$

If we consider this equation for the whole circuit then matrix form of the above equation becomes $[V_b] = [Z_b][I_b] + [V_s]$ where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_s]$ is the column vector of source voltage. Kirchhoff's Voltage Law states that the algebraic sum of voltages (or voltage drops) in any closed path of network that is traversed in a single direction is zero. Its matrix form is given by

$$\begin{aligned} [B][V_b] &= 0 \\ \Rightarrow [B]([Z_b][I_b] + [V_s]) &= 0 \\ \Rightarrow [B][Z_b][I_b] &= -[B][V_s] \end{aligned}$$

Also the branch current matrix equation is $[I_b] = [B^T][I_L]$, where $[I_L]$ is the loop current matrix and $[B^T]$ is the transpose of $[B]$.

$$\text{So, we get } [B][Z_b][B^T][I_L] = -[B][V_s]$$

4. Analysis of Graph Theoretic Approach Using Superposition Theorem :

The case of a circuit with unknown resistances and voltages is discussed below with the help of

Superposition theorem.

4.1 Superposition Theorem of Electrical Circuit¹⁰:

If a number of voltage or current sources are acting simultaneously in a linear network then the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

Consider the circuit with four unknown resistances and three unknown voltages (all are non-zero) as shown in the figure.

Figure 2: Circuit with unknown resistances and voltages

We will find the current through resistance ' r_3 ' by matrix equilibrium equation and then it will be verified by Superposition theorem.

4.2 Graph Theoretic Approach to Find the Currents :

The graph of the circuit given in Figure 2

Figure 3: Graph of the circuit

Here we get the following matrices

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad Z_b = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_2 & 0 & 0 \\ 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & r_4 \end{bmatrix},$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_L = \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix}, \quad V_s = \begin{bmatrix} v_1 \\ v_2 \\ 0 \\ v_3 \end{bmatrix}$$

So putting the values in

$$[B][Z_b][B^T][I_L] = -[B][V_s] \quad \text{we have}$$

$$\Rightarrow \begin{bmatrix} r_1 + r_2 & -r_2 & -r_2 \\ -r_2 & r_2 + r_3 & r_2 \\ -r_2 & r_2 & r_2 + r_4 \end{bmatrix} \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix} = \begin{bmatrix} v_2 - v_1 \\ -v_2 \\ -(v_2 + v_3) \end{bmatrix}$$

$$\Rightarrow X I_L = Y$$

Solving this we have the loop currents as

$$I_{L_1} = \frac{-v_1(r_2r_3 + r_2r_4 + r_3r_4) + v_2r_3r_4 - v_3r_2r_3}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} A \text{ (Ampere)}$$

$$I_{L_2} = \frac{-v_1r_2r_4 - v_2r_1r_4 + v_3r_1r_2}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} A$$

$$I_{L_3} = \frac{-v_1r_2r_3 - v_2r_3r_1 - v_3(r_1r_2 + r_1r_3 + r_2r_3)}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} A$$

$$\text{Where } r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4 \neq 0$$

Again from

$$[I_b] = [B^T][I_L], \text{ the branch currents } i_k, k = 1, 2, 3, 4 \text{ are given by}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{-v_1(r_2r_3 + r_2r_4 + r_3r_4) + v_2r_3r_4 - v_3r_2r_3}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} \\ \frac{v_1r_3r_4 - v_2(r_1r_3 + r_1r_4 + r_3r_4) - v_3r_3r_1}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} \\ \frac{-v_1r_2r_4 - v_2r_4r_1 + v_3r_1r_2}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} \\ \frac{-v_1r_2r_3 - v_2r_3r_1 - v_3(r_1r_2 + r_1r_3 + r_2r_3)}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} \end{bmatrix}$$

So, we see that $i_{r_3} = i_3 = \frac{-v_1r_2r_4 - v_2r_4r_1 + v_3r_1r_2}{r_1r_2r_3 + r_1r_2r_4 + r_1r_3r_4 + r_2r_3r_4} A$

where ' i_{r_3} ' is the current through the resistance ' r_3 ' and ' i_3 ' is the branch current of third branch.

4.3 Verification of the Results by Superposition Theorem :

Now we will verify the above result with the help of Superposition theorem. We will find the current through resistor ' r_3 '.

From the given circuit, at first we consider the voltage source ' v_1 ' only. Other two voltage sources are removed by short circuit. Then the circuit becomes

Figure 4: Circuit with voltage source ' v_1 ' only

Let us consider the node voltage at A be ' V_A '. Then from Kirchhoff's Current Law we have

$$\frac{V_A - v_1}{r_1} + \frac{V_A}{r_2} + \frac{V_A}{r_3} + \frac{V_A}{r_4} = 0$$

$$\Rightarrow V_A = \frac{v_1 r_2 r_3 r_4}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} V \text{ (Volt)}$$

So current through resistance ' r_3 ' in this case

$$i_1 = \frac{V_A}{r_3} = \frac{\frac{r_2 r_3 r_4 v_1}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4}}{r_3} A = \frac{v_1 r_2 r_4}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} A$$

Now considering the voltage source ' v_2 ' only and short circuiting the other two sources we have

Figure 5: Circuit with voltage source ' v_2 ' only

So, from Kirchhoff's Current Law at node A, we have

$$\frac{V_A}{r_1} + \frac{V_A - v_2}{r_2} + \frac{V_A}{r_3} + \frac{V_A}{r_4} = 0$$

$$\Rightarrow V_A = \frac{v_2 r_1 r_3 r_4}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} V$$

So current through resistance ' r_3 ' in this case

$$i_2 = \frac{V_A}{r_3} = \frac{\frac{r_1 r_3 r_4 v_2}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4}}{r_3} A = \frac{v_2 r_4 r_1}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} A$$

Now considering the voltage source ' v_3 ' only and short circuiting the other two sources we have

Figure 6: Circuit with voltage source ' v_3 ' only

Again from Kirchhoff's Current Law at node A we have

$$\frac{V_A}{r_1} + \frac{V_A}{r_2} + \frac{V_A}{r_3} + \frac{V_A + v_3}{r_4} = 0$$

$$\Rightarrow V_A = \frac{-v_3 r_1 r_2 r_3}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} V$$

So current through resistance 'r₃'

$$i_3 = \frac{V_A}{r_3} = \frac{\frac{-v_3 r_1 r_2 r_3}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4}}{r_3} A = \frac{-v_3 r_1 r_2}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} A$$

So by Superposition theorem total current through resistor 'r₃'

$$i_{r_3} = i_1 + i_2 + i_3 = \frac{v_1 r_2 r_4 + v_2 r_4 r_1 - v_3 r_1 r_2}{r_1 r_2 r_3 + r_1 r_2 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4} A, \text{ which is same as}$$

the current obtained in matrix form through resistance 'r₃'.

In case of matrix method negative sign is just showing that current is in opposite direction.

4.4 Generalisation of the Problem :

If an electrical circuit contains 'n' branches each branch containing resistance 'r_i' and voltage 'v_i' for i= 1, 2, 3....n respectively then the circuit becomes

Figure 7: Circuit containing 'n' resistances and 'n' voltages

The graph of the circuit is given by

Figure 8: Graph of the circuit

From this we can find B , Z_b , B^T , I_L and V_S for the circuit containing 'n' branches and (n-1) loops such that

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & -1 & 0 & \dots & 0 \\ 0 & -1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -1 & 0 & 0 & \dots & -1 \end{bmatrix}_{(n-1) \times n}$$

$$Z_b = \begin{bmatrix} r_1 & 0 & 0 & \dots & 0 \\ 0 & r_2 & 0 & \dots & 0 \\ 0 & 0 & r_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & r_n \end{bmatrix}_{n \times n}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & -1 & -1 & \dots & -1 \\ 0 & -1 & 0 & \dots & 0 \\ 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -1 \end{bmatrix}_{n \times (n-1)}$$

$$I_L = \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \\ \vdots \\ \vdots \\ \vdots \\ I_{L_{n-1}} \end{bmatrix}_{(n-1) \times 1}$$

$$V_S = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ \vdots \\ \vdots \\ v_n \end{bmatrix}_{n \times 1}$$

So putting these values in

$$[B][Z_b][B^T][I_L] = -[B][V_s] \text{ and after simplification we get}$$

$$XI_L = Y \text{ where } X = [B][Z_b][B^T] \text{ and } Y = -[B][V_s] \text{ such that}$$

$$X = \begin{bmatrix} r_1 + r_2 & -r_2 & -r_2 & -r_2 & \cdot & \cdot & \cdot & -r_2 \\ -r_2 & r_2 + r_3 & r_2 & r_2 & \cdot & \cdot & \cdot & r_2 \\ -r_2 & r_2 & r_2 + r_4 & r_2 & \cdot & \cdot & \cdot & r_2 \\ -r_2 & r_2 & r_2 & r_2 + r_5 & \cdot & \cdot & \cdot & r_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -r_2 & r_2 & r_2 & r_2 & \cdot & \cdot & \cdot & r_2 + r_n \end{bmatrix}_{(n-1) \times (n-1)}$$

$$Y = \begin{bmatrix} -(v_1 + v_2) \\ v_2 + v_3 \\ v_2 + v_4 \\ \cdot \\ \cdot \\ \cdot \\ v_2 + v_n \end{bmatrix}_{(n-1) \times 1}$$

From this we can find the loop current matrix $I_L = X^{-1}Y$.

So we see that we can apply graph theoretic approach to find the currents in a circuit containing 'n' branches with 'n' resistances and 'n' voltages which is difficult in Superposition theorem. So using graph theory and matrix we can study a complex circuit.

4.5 New Results

In $X I_L = Y$ we see that

- 1) The matrix X is symmetric.

- 2) The diagonal elements of X are given by the sum of the resistances of loop L_i , $i=1, 2, 3, \dots, n-1$.
- 3) To find the matrix X in $X I_L = Y$ we have

$a_{ii} = \text{Sum of the resistances in } i^{\text{th}} \text{ loop}$

$a_{1i} = a_{i1} = -(\text{Resistance in the tree branch}), i \neq 1$

and for remaining elements

$a_{ij} = a_{ji} = \text{Resistance in the tree branch}$

- 4) To find the column matrix Y in $X I_L = Y$

$$a_{li} = - \begin{cases} \text{sum of the voltages in the loop if they are connected in same way} \\ \text{with the sign of the bigger voltage in the loop.} \\ \text{difference of the voltages in the loop if they are connected in opposite way} \\ \text{with the sign of the bigger voltage in the loop.} \end{cases}$$

4.6 Application of the Model:

Our matrix model will help to find the loop currents and branch currents of electrical circuits containing 'n' branches. It is laborious when we apply some traditional processes such as nodal analysis and some well known theorems of electrical circuits to find the branch currents containing many loops or branches. Our model will overcome this problem. In future it may be helpful to develop more complex circuit solving programs.

4.7 Conclusion

From the above study we observed that the result obtained by graph theoretic approach is same as the result obtained by Superposition Theorem. So the graph theoretic approach is verified by existing Superposition Theorem. The value of currents can be obtained for different value of ' r_i ' and ' v_i ' by network equilibrium equation without knowing the nodal and mesh analysis of electrical circuits.

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