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Excluded Energy and Relative Encircled Energy plays an important role in the Optical Systems as Point-Image Quality-Assessment Parameters

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Abstract

Encircled Energy Factor (EEF), Excluded Energy and Relative Encircled Energy are important corollaries of the Point Spread Function (PSF) of an optical system. In this paper, we have studied the Excluded Energy, Relative Encircled energy to understand their role as Point Image quality assessment parameters of Optical systems with amplitude apodised rotational symmetric filter functions.

Key-words: Mathematical Optics, Apodisation, Encircled Excluded Energy Factor, Relative Encircled Energy, Pupil Function etc..

1. Introduction

Encircled energy is an important parameter which can be studied as a measure of image quality. According to Wetherill[1], **"Excluded energy"** is the energy difference between total energy and the energy concentrated within a specified circle of radius δ. It is denoted with Ex [δ] and is also useful to study the structure of a specified ring in detail. It is also known as the **"Dispersion Factor"**, as designated by Dossier[2], higher value of this factor indicates better functioning of the pupil function. The most desirable pupil function is that which can minimize the excluded energy and maximize the encircled energy. Surender[3] has remarked on the importance of the excluded energy as follows.

"The contrast at the centre of the image of a black disc, seen against a uniform incoherent background is what can be interpreted as the physical significance of $Ex(\delta)$**".** If the energy in the outer rings of the diffraction pattern is to be examined in details, it is convenient to focus more on the excludedenergy rather the encircled energy. This aspect has made the study on $Ex(\delta)$ more useful, in various techniques of apodisation, to change the ring structures in a pre-determined way. The study of $Ex(\delta)$ is also useful in several photometric analysis. Relative encircled energy the most important corollary of the Point Spread Function (*PSF*) is the **"Relative Encircled Energy".** It measures the fraction of the total energy in the *PSF*, which lies within a specified radius 'δ' in the plane of observation or detection. We will designate this important parameter by the symbol $REE(\delta)$. Lord Rayleigh[4] was the first to point out the importance of the encircled energy factor to find the illuminations in the various rings of the diffraction pattern and presented a formula for calculating the same.

When a converging spherical wave is diffracted by a circular aperture the classical theory of focusing predicts that light energy is highly concentrated in the geometrical focal plane. It means that there is a maximum amount of energy within a receiving circle of a given radius centered at the aperture axis and placed in the geometrical focal plane which contains more energy per unit area than any other plane parallel to it. Thus, it comes out that the $EEF(\delta)$ is the primary corollary of the and is the factor, which describes the integrated behavior of the point source diffraction image. It is a sensible image quality evaluation parameter of an optical system, with this phenomena Excluded energy Factor, Relative Encircled energy are very close to Encircled energy we have studied in this paper these two parameters. Murthy[5] was investigated the PSF Based corollaries viz Encircled energy Excluded energy and energy increment in the presence of defocusing by employing co-sinusoidal amplitude filters. Karunasagar[6] has studied the Encircled energy Relative encircled energy and Excluded energy, Displaced energy and zonal increment for rotationally symmetric Ratnam[7] similar studies have been carried out for multiple coded apertures (MACA) and complimentary multiple annuli coded apertures (CMACA). Steal[8] and Mehatha[9] have studied the encircled energy in the Fraunhoper diffraction pattern with circular apertures with triangular apodisation filters Surender, Seshagiri Rao and Mondal[10] have studied the encircled energy and its complimentary quantity, Excluded energy using Lanczos apodisation filters. Keshavulu[11] was investigated the encircled energy in the presence of individual and the combined effects of defocusing and primary spherical aberration in the case of optical systems apodised by shaded-aperture systems. V.N Mahajan[12] have obtained a closed form solution for the excluded power, using the well-known recurrence relations of Bessel functions of the first kind.

P. Thirupathi[13] has been investigated Encircled energy factor in impulse response functions of optical systems with first-order parabolic filters, that is $f(r) = (\alpha + \beta r^2)$. In our investigation we are using filter function $f(r) = (1 - \beta r)$ and get results for Excluded energy Factor, Relative encircled energy, we have discussed the results with figures and tabular. This paper organized as fallows. In section 2, mathematical expressions for Excluded energy Factor and Relative Encircled energy, in section3, given results and discussions. We have given important conclusions in section4 finally, suggested applications given in section5.

2. Mathematical formulations:

Excluded Energy can be represented mathematically as

$$Ex(\delta) = 1 - EEF(\delta) \tag{2.1}$$

Where EEF (δ) is Encircled Energy Factor it is denoted as

$$EEF(\delta) = \frac{\int_0^{\delta} |G_F(0,z)|^2 \, zdz}{\int_0^{15} |G_F(0,z)|^2 \, zdz} \tag{2.2}$$

Where $G(0, z)$ is the amplitude in the image plane at a point z, it is defined[14] as

$$G(0,Z) = 2\int_0^1 f(r) J_0(Zr) r \, dr \tag{2.3}$$

Where $f(r)$ is the pupil function which defines the nature of transmission over the pupil of the aperture of the optical system the pupil-function $f(r)$ can be mathematically represented as

$$f(r) = (1 - \beta r) \tag{2.4}$$

Where β is the apodisation parameter which controls the amplitude transmission of the pupil and r is the normalized distance of a point on the pupil from its centre and $J_0(Zr)$ is the Bessel function of the first kind with zero order for the argument (Zr) finally from equation (2.1) and (2.2) we get the mathematical expression for Ex[δ]. **Relative Encircled energy** is defined as the ratio of the light energy within a specified circle of radius δ centered on the diffraction head due to the non-airy pupil to the total light energy in the diffraction pattern due to the Airy-pupil. Mathematically, it can be represented as:

$$REE(\delta) = \frac{\int_0^{\delta} |G_F(0,Z)|^2 \, zdz}{\int_0^{\infty} |G_A(0,z)|^2 \, zdz} \tag{2.5}$$

Where the subscripts A and F stand for Airy filtered and non-airy pupils respectively. It has been observed that for rotationally symmetric pupil functions,

For Airy pupil function the apodisation parameter β values are difference from zero, Non-airy pupil function the apodisation parameter β values are equal to zero in this case the numerator function $G_F(0,z)$ is defined as below,

$$G_F(0,z) = 2\int_0^1 J_0(zr) r dr \tag{2.6}$$

Substituting amplitude functions in both cases airy pupil and non-airy pupil in equation (2.5) then we get corresponding results for Relative Encircled energy.

Results and Discussions

Excluded Energy Factor: We have used the expression $Ex(\delta) = 1 - EEF(\delta)$ for evaluating the excluded energy $Ex(\delta)$. The results have been shown in the figure 1 and the table 1 for various values of the apodisation parameter β. For lower values of δ in the range $0 \le \delta \le 4$, the Ex [δ] decreases very rapidly. From δ>4.0 upto12.0, the factor indicates that the energy displacement is outward while the negative sign indicates that the energy displacement is inward. This factor is useful to compare the energy distribution in the case of actual optical imaging systems to that of perfect systems. This is a more sensitive quality factor in the case of central obscuration in the aperture.

Relative Encircled Energy Facor REE(δ): We used the equation (2.5)

For evaluating the Relative Encircled energy, the results have been shown in figure 2 and the table 2. For non-Airy Pupil function that is apodisation parameter β=0 for various values of δ gives same value of Relative Encircled Energy, for high value of δ that is δ=15 with various values of β then the Relative Encircled energy become identical it is evident for convergence. For lover values of δ in the range $1 \leq \delta \leq 3$ the Relative Encircled energy decreases up to 0.67. For higher value of δ in the range $4 \leq \delta \leq 14$ then the Relative Encircled energy increases and instantly small decreasing after that Relative encircle energy become identical and it goes to infinite.

Conclusions

The pupil function $f(r) = (1 - \beta r)$ is rotationally symmetric. For higher values of δ with various values of β then the Excluded energy factor and Relative encircled energy both increases monotonically and becomes identical it happens with the effect of **our** consideration filter function and Bessel function.

Suggested Applications:

In Microscopy: Due to the difficulty of finding resolution of the point like radiating sources, experimental determination of the shape of PSF in microscopy, is usually tricky. The most compact diffraction-limited shape of the PSF is desirable, if not essential. To achieve this goal, theoretical models presented in this thesis, make the detailed calculations of the PSF under various imaging conditions, possible. Further, by using appropriate optical elements, e.g., a spatial light modulator, the shape of the PSF can be engineered towards different applications.

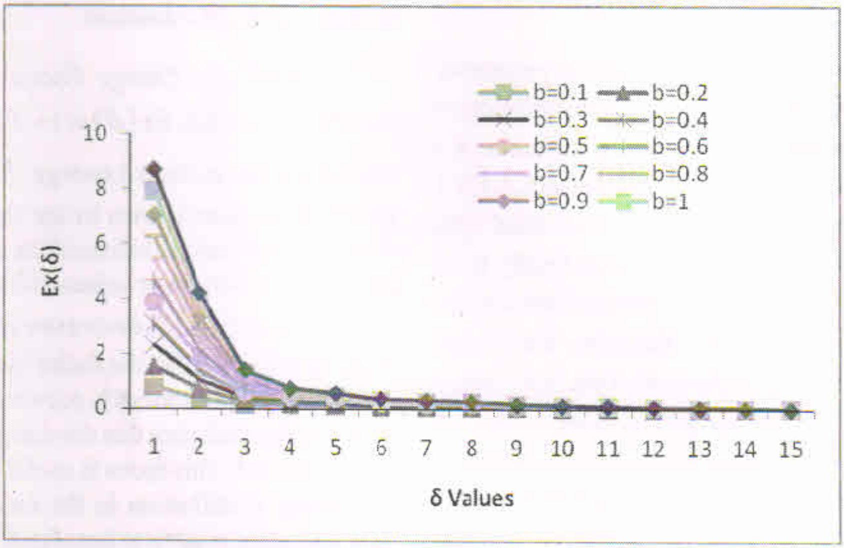

Fig. 1. Excluded Energy Factor with δ various values of β

Table 1

δ	β=0	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6	β=0.7	β=0.8	β=0.9	β=1
1	0.76943	0.76987	0.77073	0.77217	0.77447	0.77801	0.78337	0.7914	0.80329	0.82064	0.84521
2	0.3555	0.35337	0.35189	0.35145	0.35267	0.35652	0.36448	0.37882	0.40297	0.44175	0.50118
3	0.14651	0.13767	0.12864	0.11972	0.11142	0.10466	0.10094	0.10283	0.11449	0.1424	0.19575
4	0.12511	0.11343	0.10076	0.08712	0.07265	0.05775	0.04319	0.03055	0.02268	0.02459	0.04426
5	0.10084	0.09303	0.08435	0.07471	0.06407	0.05245	0.04006	0.02743	0.01577	0.00743	0.0066
6	0.05949	0.05542	0.05087	0.04577	0.04007	0.03376	0.0269	0.01967	0.0126	0.00676	0.00416
7	0.04991	0.04638	0.04244	0.03804	0.03315	0.02774	0.02189	0.01579	0.0099	0.0052	0.00347
8	0.04411	0.0411	0.03772	0.03394	0.02972	0.02505	0.01995	0.01459	0.00934	0.005	0.00304
9	0.02722	0.02529	0.02313	0.02073	0.01807	0.01515	0.01201	0.00877	0.0057	0.00335	0.00275
10	0.02099	0.01929	0.01741	0.01532	0.01303	0.01054	0.0079	0.00525	0.00286	0.00127	0.00141
11	0.0191	0.01761	0.01596	0.01412	0.01209	0.00985	0.00744	0.00495	0.0026	0.0008	0.00033
12	0.01037	0.00963	0.00879	0.00786	0.00681	0.00565	0.00439	0.00306	0.00175	0.00065	0.00014
13	0.00574	0.00531	0.00482	0.00429	0.00369	0.00303	0.00232	0.00158	0.00087	0.0003	9.8E-05
14	0.005	0.00463	0.00422	0.00376	0.00324	0.00267	0.00206	0.0014	0.00077	0.00024	1.2E-05
15	1.1E-16	0	0	1.1E-16	0	0	0	0	0	0	1.1E-16

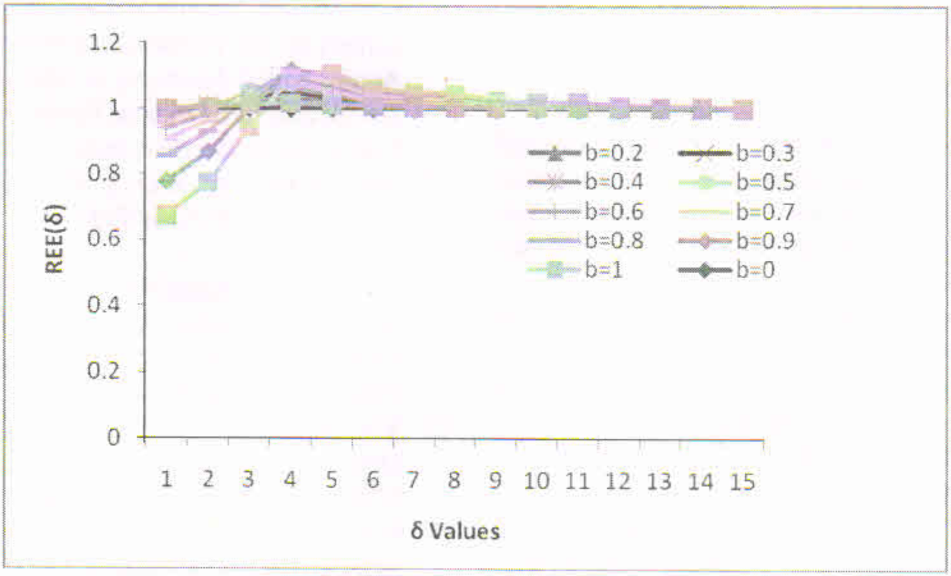

Fig. 2. Relative Encircled Energy with δ various values of β

Table 2

1	0.76943	0.76987	0.77073	0.77217	0.77447	0.77801	0.78337	0.7914	0.80329	0.82064	0.84521
d	b=0	b=0.1	b=0.2	b=0.3	b=0.4	b=0.5	b=0.6	b=0.7	b=0.8	b=0.9	b=1
1	1	0.9981	0.9944	0.9881	0.9781	0.9628	0.9395	0.9047	0.8531	0.7779	0.6713
2	1	1.0033	1.0056	1.0063	1.0044	0.9984	0.9861	0.9638	0.9263	0.8662	0.774
3	1	1.0104	1.0209	1.0314	1.0411	1.049	1.0534	1.0512	1.0375	1.0048	0.9423
4	1	1.0134	1.0278	1.0434	1.06	1.077	1.0936	1.1081	1.1171	1.1149	1.0924
5	1	1.0087	1.0184	1.0291	1.0409	1.0538	1.0676	1.0816	1.0946	1.1039	1.1048
6	1	1.0043	1.0092	1.0146	1.0207	1.0274	1.0347	1.0423	1.0499	1.0561	1.0588
7	1	1.0037	1.0079	1.0125	1.0176	1.0233	1.0295	1.0359	1.0421	1.0471	1.0489
8	1	1.0032	1.0067	1.0106	1.0151	1.02	1.0253	1.0309	1.0364	1.0409	1.043
9	1	1.002	1.0042	1.0067	1.0094	1.0124	1.0156	1.019	1.0221	1.0245	1.0252
10	1	1.0017	1.0037	1.0058	1.0081	1.0107	1.0134	1.0161	1.0185	1.0202	1.02
11	1	1.0015	1.0032	1.0051	1.0072	1.0094	1.0119	1.0144	1.0168	1.0187	1.0191
12	1	1.0008	1.0016	1.0025	1.0036	1.0048	1.006	1.0074	1.0087	1.0098	1.0103
13	1	1.0004	1.0009	1.0015	1.0021	1.0027	1.0034	1.0042	1.0049	1.0055	1.0057
14	1	1.0004	1.0008	1.0013	1.0018	1.0023	1.003	1.0036	1.0043	1.0048	1.005
15	1	1	1	1	1	1	1	1	1	1	1

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