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Heat source effect on unsteady MHD flow through porous medium with transpiration of heat and mass transfer past a porous vertical moving plate

HARISH CHANDRA SHARMA, P. C. GUPTA and N. K. VARSHNEY

Department of Mathematics, S. V. College, Aligarh U. P. (INDIA)

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Abstract

A solution of the problem to study the effect of heat source on unsteady MHD free convection flow of an incompressible, viscous, electrically conducting fluid through porous media past an infinite porous vertical non conducting moving plate in the presence of uniform transverse magnetic field and transpiration is investigated. This type of problem finds application in many technological and engineering fields such as rocket propulsion systems, space craft re-entry aerothermodynamics, cosmical flight aerodynamics, plasma physics, Glass production and furnace engineering. Velocity, temperature and concentration of the flow have been presented for various parameters. In this study velocity of fluid increases with the increase in G_m (Modified Grashof number), K (Porosity parameter) and S (Heat source parameter), but it decreases with the increase in M (Hartmann number).

Key words : Heat source, Transpiration, Viscous fluid, Unsteady, MHD flow, Porous Medium, Heat and Mass transfer.

Introduction

Thermal boundary layer flow problems are classified into two categories e.g., (i) free/natural convection flow and (ii) forced convection flow and have many applications in the areas of industries and engineering. The Grashof number, Prandtl number, Hertmann number and porosity play an important role on free convection flows. The problem of free convection flows past a porous/non porous vertical plate has been considered by many researchers, e.g., Schlichting[17], Gupta[8], Soundalgekar[20], Mishra and Mohapatra[12]. Soundalgekar and Gupta[22], Bansal[1], Soundalgekar[21], Georgantopoulos[7], Rapits and Tzivanidis[16] Mohapatra and Senpati[13], Sharma[18] and Sharma and Mishra[19]. Jadon *et al.*[10] studied effects of mass transfer on unsteady MHD flow through porous medium and heat transfer past a porous vertical moving plate.

In the context of space technology and

in processes involving high temperatures the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, and power plants for inter planetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of radiative transfer in these process. The interaction of radiation with laminar free convection heat transfer from a vertical plate was investigated by Cess[2] for an absorbing, emitting fluid in the optically thick region, using the singular perturbation technique. Vedat *et al.*[24] considered a similar problem in both the optically thick regions and used the approximate integral technique and first-order profiles to solve the energy equation. Cheng and Ozisik[4] considered a related problem for an absorbing, emitting and isotropically scattering fluid, and treated the radiation part of the problem exactly with the normal-mode expansion technique. Raptis[14] has analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. Hossain and Takhar[9] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. In all these papers the flow is considered to be study. The unsteady flow past a moving plate in the presence of free convection and radiation were studied by Mansour[11]. Raptis and Perdikis[15] studied the effects of thermal radiation and free convective flow past moving plate. Das *et al.*[5] have analyzed the radiation effects on flow past an impulsively started infinite isothermal vertical plate. Chamkha *et al.*[3] studied the effect of radiation on free convective flow past a semi-infinite vertical plate with mass transfer. Genesan and Loganadhan[6] studied the radiation and mass transfer effects on flow of incompressible viscous fluid past moving vertical cylinder using Rosseland approximation. Vir *et al.*[25] have discussed effect of radiation on unsteady MHD flow through porous medium with mass and heat transfer past a porous vertical plate. Recently, Varshney *et al.*[23] studied transpiration effect on unsteady MHD flow through porous medium with heat and mass transfer past a porous vertical moving plate.

In the present paper we consider the problem Varshney *et al.*[23] with heat source. The purpose of this study is to investigate the effect of heat source on unsteady MHD free convection flow through porous media past an infinite porous vertical non-conducting moving plate in the presence of uniform magnetic field and transpiration.

Formulation of the problem :

Let us consider free convective flow of a viscous fluid of small electrical conductivity through a porous medium bounded by an oscillating infinite plate in slip flow regime with heat source/sink in the presence of uniform magnetic field, the velocity components u, v in direction of x, y respectively, temperature by T and concentration by C. A uniform magnetic field B_o is acting along the y-axis. Under such conditions the induced magnetic field due to the flow may be neglected with respect to the applied field, the pressure p in the fluid is assumed constant. If Vo represents the velocity of suction or injection at the plate, the equation of continuity is:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

with the condition y = 0, v leads to the result $v = -V_o$

The boundary layer equations are:

$$\frac{\partial u}{\partial t} - V_o \frac{\partial u}{\partial y} = \frac{dU(t)}{dt} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$+g\beta^*(C - C_\infty) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K^*}\right)\{u - U(t)\} \quad (2)$$

$$\frac{\partial T}{\partial t} - V_o \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + S^*(T - T_\infty) \quad (3)$$

$$\frac{\partial C}{\partial t} - V_o \frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial y^2}\right) + D_1\left(\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β^* is the coefficient of concentration expansion, ν is the Kinematic viscosity, T_∞ and C_∞ are the temperature and concentration of the fluid in the free stream, σ is the electric conductivity, B_o is the magnetic induction, K is coefficient of porosity, α is the thermal conductivity, D is concentration diffusivity, D_1 is thermal diffusivity, U(t) is the main stream velocity, q_r is the radiative heat flux, S* is the coefficient of heat source.

By using Rosseland approximation for the radiation we take

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (5)$$

where σ^* the Stefan - Boltzmann constant and k* the mean absorption coefficient.

We assume that the temperature differences within the flow are such that T^4 may be expressed as a linear function of temperature.

This is accomplished by expanding T^4 in a Taylor series about and neglecting higher-order terms, thus

$$T^4 \simeq 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

The boundary conditions of the problem are

$u = V_o, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \quad (7)$

$u \to U(t), \quad T \to T_\infty, \; C \to C_\infty \quad \text{as} \quad y \to \infty$

On introducing the following non dimensional quantities

$$u^* = \frac{u}{V_0}(\text{Velocity}),\; t = \frac{tV_o^2}{\nu},\; y^* = \frac{yV_o}{\nu},$$

$$\theta^* = \frac{(T - T_\infty)}{(T_w - T_\infty)},\; \phi^* = \frac{(C - C_\infty)}{(C_w - C_\infty)},$$

$$U^*(t^*) = \frac{U(t)}{V_0}$$

Equations (2), (3) and (4) after dropping the asterisks (*) can be written as

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{dU(t)}{dt} + \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - Q\{u - U(t)\} \quad (8)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{N_1}\frac{\partial^2 \theta}{\partial y^2} + S\theta \quad (9)$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + A\frac{\partial^2 \theta}{\partial y^2} \quad (10)$$

where

$$K = \frac{KV_o^2}{\nu^2} \text{ (Porosity Parameter)},$$

$$P_r = \frac{\nu}{\alpha} \text{ (Prandtl number)},$$

$$S_c = \frac{\nu}{D} \text{ (Schimdt number)},$$

$$M = \frac{\sigma B_o^2 \nu}{\rho V_o^2} \text{ (Hartmann number)},$$

$$G_r = \frac{g\beta\nu(T_w - T_\infty)}{V_o^3} \text{ (Grashof number)},$$

$$G_m = \frac{g\beta^*\nu(C_w - C_\infty)}{V_o^3} \text{ (Modified Grashof number)}$$

$$N_1 = \left(\frac{3NP_r}{3N+4}\right)$$

$$N = \left(\frac{k^*k}{4\sigma^* T_\infty^3}\right) \text{ (Radiation parameter)}$$

$$A = \frac{D_1(T_w - T_\infty)}{\nu(C_w - C_\infty)} \text{ (Thermal Diffusion parameter)}$$

$$S = \frac{S^*}{\nu} \text{ (Heat source parameter)}$$

$$Q = M + \frac{1}{K}$$

with the boundary conditions :

$$u = 1 \quad \theta = 1, \quad \phi = 1 \quad \text{at } y = 0 \qquad (11)$$
$$u \to U(t),\ \theta \to 0 \quad \phi \to 0 \quad \text{as } y \to \infty$$

Solution of the problem :

The solutions of equations (8), (9) and (10) are :

$$u = u_o + \varepsilon u_1 e^{i\omega t}$$
$$\theta = \theta_o + \varepsilon \theta_1 e^{i\omega t}$$
$$\phi = \phi_o + \varepsilon \phi_1 e^{i\omega t}$$
$$U(t) = 1 + \varepsilon e^{i\omega t} \qquad (12)$$

Using equations (12) in equations (8), (9) and (10) and separating harmonic and non-harmonic terms, we get

for u: $$u_o'' + u_o' - Qu_o = -G_r\theta_o - G_m\phi_o - Q \qquad (13)$$

$$u_1'' + u_1' - (Q + i\omega)u_1 = -G_r\theta_1 - G_m\phi_1 - (Q + i\omega) \qquad (14)$$

for : $$\theta_o'' + N_1\theta_o' + N_1 S\theta_o = 0 \qquad (15)$$

$$\theta_1'' + N_1\theta_1' + (S - i\omega)N_1\theta_1 = 0 \qquad (16)$$

for : $$\phi_o'' + S_c\phi_o' = -AS_c\theta_o'' \qquad (17)$$

$$\phi_1'' + S_c\phi_1' - i\omega S_c\phi_1 = -AS_c\theta_1'' \qquad (18)$$

where dash denotes differentiation with respect to y.

with the boundary conditions

$u_o=1$, $u_1= 0$, $\theta_o= 1$, $\theta_1= 0$, $\phi_o= 1$, $\phi_1= 0$ at $y = 0$ (19)

$u_o \to 1$, $u_1 \to 1$, $\theta_o \to 0$, $\theta_1 \to 0$, $\phi_o \to 0$, $\phi_1 \to 0$ as $y \to \infty$

The solution of the equations (13) to (18) applying the boundary conditions (19) is

$$u_o = 1 + \{L_1G_r + L_2G_m + A_4G_m(L_2 - L_1)\}e^{-A_1 y} - L_1G_r e^{-A_2 y} - G_mL_2(1+A_4)e^{-S_c y} + A_4L_1G_m e^{-A_2 y} \qquad (20)$$

$$u_1 = F_1(y) + iF_2(y) \qquad (21)$$

$$\theta_o = e^{-A_2 y} \qquad (22)$$

$$\theta_1 = 0 \qquad (23)$$

$\phi_o = (1 + A_4)e^{-S_c y}$ (24)

$\phi_1 = 0$ (25)

where

$$A_1 = \frac{[1+\{1+4Q\}^{1/2}]}{2}$$

$$A_2 = \frac{[N_1+\{N_1^2 - 4N_1S\}^{1/2}]}{2}$$

$$L_1 = \frac{1}{A_2^2 - A_2 - Q}$$

$$L_2 = \frac{1}{S_c^2 - S_c - Q}$$

$$A_3 = \frac{1+\alpha_1}{2},$$

$$B_3 = \frac{\beta_1}{2},$$

$$A_4 = \frac{AA_2S_c}{A_2 - S_c}$$

$$\alpha_1 = \left[\frac{\{Q_1^2 + \omega_1^2\}^{1/2} + Q_1}{2}\right]^{1/2}$$

$$\beta_1 = \left[\frac{\{Q_1^2 + \omega_1^2\}^{1/2} + Q_1}{2}\right]^{1/2}$$

$Q_1 = 1 + 4Q,$

$\omega_1 = 4\omega$

$$F_1 = 1 - e^{-A_3 y}\cos(B_3 y),$$

$$F_2 = e^{-A_3 y}\sin(B_3 y)$$

Hence, the expression for the velocity, temperature and concentration are given by

$u = u_o + \varepsilon[F_1\cos(\omega t) + F_2\sin(\omega t)]$ (26)

$\theta = e^{-A_2 y}$ (27)

$\phi = (1 + A_4)e^{-S_c y}$ (28)

We can calculate the expression for the skin friction, the rate of heat transfer and rate of concentration transfer as

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = -A_1\{L_1G_r + L_2G_m + A_4G_m(L_2-L_1)\} + A_2L_1G_r + S_cG_mL_2(1+A_4) - A_4A_2L_1G_m + \varepsilon[A_3\cos(\omega t) - B_3\sin(\omega t)]) \quad (29)$$

$$q_1 = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -A_2 \quad (30)$$

$$q_2 = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -(1 + A_4)S_c + A_2A_4 \quad (31)$$

Results and Discussion

Fluid Velocity Profile of boundary layer flow is tabulated in Table 1 and plotted in Fig. 1 having six Graphs for $\varepsilon = 0.25$, $G_r = 5$, $P_r = 0.71$, $S_c = 0.4$, $\omega t = \pi/6$, $N = 1$, $\omega = 1$, $A = 1$ and following different values of G_m, M, K and S.

	G_m	M	K	S
For Graph-I	3	2	2	0
For Graph-II	3	2	2	0.06
For Graph-III	3	2	2	0.03
For Graph-IV	5	2	2	0.03
For Graph-V	3	6	2	0.03
For Graph-VI	3	2	4	0.03

Table 1. Values of velocity at ε=0.25, G_r = 5, P_r = 0.71, S_c = 0.4, ωt = $\pi/6$, N = 1, ω = 1, A = 1 and different values of G_m, M, K and S.

y	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1	3.07348	3.10900	3.17846	3.57744	2.05866	3.27224
2	2.82892	2.88143	2.98877	3.28687	1.89858	3.04447
3	2.44488	2.50542	2.63330	2.81630	1.73599	2.63603
4	2.13548	2.19821	2.33465	2.43291	1.61111	2.29848
5	1.90181	1.96286	2.09953	2.13986	1.51637	2.03922

Table 2. Values of temperature ' θ ' at P_r = 0.71, S_c = 0.4, N = 1, A = 1 and different values of S.

y	Graph-I	Graph-II
0	1.00000	1.00000
1	0.76338	0.80252
2	0.58275	0.64404
3	0.44486	0.51685
4	0.33960	0.41478
5	0.25924	0.33287

Table 3. Values of skin friction ' τ ' at ε = 0.25, G_r = 5, P_r = 0.71, S_{rc} = 0.4, ωt = $\pi/6$, N = 1, ω = 1, A = 1 and different values of G_{rm}, M, K and S.

y	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	7.07231	7.12685	7.22574	8.50994	5.13750	7.41754
1	7.03198	7.08645	7.18535	8.46955	5.10318	7.37624
2	6.97148	7.02590	7.12479	8.40899	5.03933	7.31554
3	6.89324	6.94759	7.04648	8.33068	4.94850	7.23789
4	6.80037	6.85464	6.95354	8.23774	4.83430	7.14637
5	6.69657	6.75078	6.84967	8.13387	4.70130	7.04463

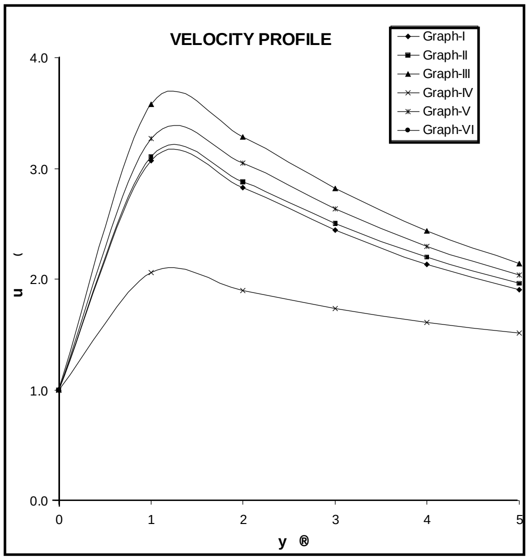

Fig. 1

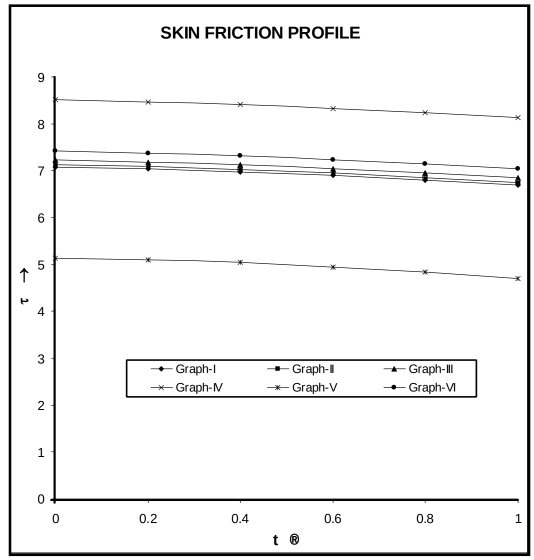

Fig. 3

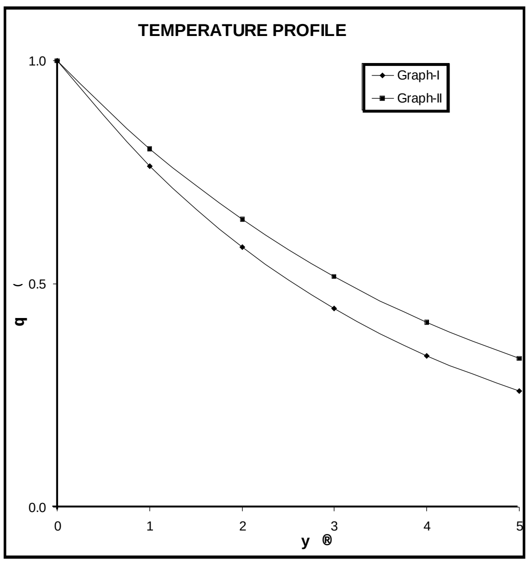

Fig. 2

It is observed from Fig. 1 that all velocity Graphs are increasing sharply up to y = 1.2, then after velocity in each Graphs begins to decrease and tends to zero with the increase in y. It is also observed from Fig. 1 that velocity increases with the increase in G_m, K and S, but it decreases with the increase in M.

Skin friction Profile is tabulated in Table 2 and plotted in Fig. 2 having six Graphs. It is observed that skin friction increases with the increase in G_m, K and S, but it decreases with the increase in M.

Temperature Profile is tabulated in Table-III and plotted in Fig. 3 having two Graphs. It is observed that temperature increases with the increase in S.

Particular case :

When S is equal to zero, this problem reduces to the problem of Varshney *et. al.*[23].

Conclusion

1. Velocity of fluid increases with the increase in S (Heat source parameter).

2. Skin friction increases with the increase in S.
3. Temperature increases with the increase in S.

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