Ultra Scientist Vol. 24(2)B, 295-300 (2012).

Effect of Hall currents on the vorticity of MHD flow of an elastico-viscous fluid past an accelerated plate

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(Acceptance Date 25th June, 2012)

Abstract

In this paper an attempt has been made to study the effect of Hall currents on the vorticity of flow of an elastico-viscous fluid with transverse magnetic field past an infinite horizontal plate for both the classes of impulsive as well as uniformly accelerated motion of the plate. The magnetic lines of force are assumed to be fixed relative to the plate. In this study vorticity increases with the increase in k (Elasticity Parameter) and m (Hall currents Parameter), but it decreases with the increase in M (Magnetic Parameter).

Key words : Elastico-viscous fluid, Hall currents, Magnetic field, An accelerated plate.

Introduction

Non-Newtonian fluids are of great importance in modern technology. Different equations have been proposed by many researchers for these fluids. A system of constitutive equations have been proposed by Walters[1] for elastico-viscous fluid, and one of them is known as Walters liquid B, which helps us to explain the flow phenomenon of certain class of Non-Newtonian fluids with short memories. The MHD flows past an impulsively started infinite plate in Walters liquid B was studied by Rawat[2]. Recently, Tyagi and Ranjan[3] have analysed on the vorticity of MHD flow of an Elastico-viscous fluid past an accelerated plate. We consider the problem of Tyagi and Ranjan[3] with Hall currents. The aim of present study to derive the effect of magnetic field, elasticity and Hall currents.

Formulation and solution of problem :

At time $t' \leq 0$, the plate, the fluid and the magnetic field are at rest. But at time $t' > 0$, the plate is accelerated with velocity $u' = Ut'^n$. Therefore the initial boundary conditions are

$$\left.\begin{array}{lll} t'\le 0, & u'=0 & \text{for all } y' \\ t'>0, & u'=Ut'^{n} & \text{for all } y'=0 \\ & u'\to 0 & \text{for all } y'\to\infty \end{array}\right\} \quad (1)$$

For a weakly conducting fluid, we neglect the induced magnetic field due to the flow with respect to it.

The equation governing the motion of the MHD flow of an elastico-viscous fluid with Hall currents is given by

$$\frac{\partial u'}{\partial t'} = \nu(n,\nu)\frac{\partial^2 u'}{\partial y'^2} - k'\frac{\partial^3 u'}{\partial y'^2\,\partial t'} - \frac{\sigma}{\rho(1+m^2)}B_0^2 u' \quad (2)$$

where ρ is density of fluid, ν is kinematic viscosity, m is Hall currents parameter, σ is electrical conductivity of the fluid, B_o is the magnetic field.

The equation (2) is valid when the magnetic lines of force are fixed relative to the fluid. Now if the magnetic field is also accelerated with the same velocity as the plate *i.e.* $u'=Ut'^{n}$, we may account for the relative motion.

The equation (2) is replaced by

$$\frac{\partial u'}{\partial t'} = \nu\frac{\partial^2 u'}{\partial y'^2} - k'\frac{\partial^3 u'}{\partial y'^2\,\partial t'} - \frac{\sigma}{\rho(1+m^2)}B_o^2(u'-Ut'^{n}) \quad (3)$$

which is valid when the magnetic lines of force are fixed relative to the fluid.

CASE 1 : IMPULSIVE START

In this case n = 0 introducing the non-dimensional quantities

$$y=\frac{Uy'}{\nu},\quad t=\frac{U^2t'}{\nu},\quad u=\frac{u'}{U} \quad (4)$$

in equation (3) we get

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}-k\frac{\partial^3 u}{\partial y^2\partial t}-\left(\frac{M}{1+m^2}\right)(u-1) \quad (5)$$

where

$$k=\frac{U^2k'}{\nu^2}\ \text{(Elasticity parameter)}$$

$$M=\frac{\sigma B_o^2\nu}{\rho U^2}\ \text{(Magnetic parameter)}$$

with boundary conditions

$$\left.\begin{array}{lll} \le 0, & u=0 & \text{for all } y \\ t>0, & u=1 & \text{for all } y=0 \\ & u\to 0 & \text{for all } y\to\infty \end{array}\right\} \quad (6)$$

The solution of equation (5) using Laplace transform technique is given by Rawat[2], as

$$u=1-e^{-M_1t}\operatorname{erf}\left(\frac{y}{2\sqrt{t}}\right)+k\left[\frac{y}{4\sqrt{\pi t^3}}\left(1-\frac{y^2}{2t}\right)e^{-(M_1t+y^2/4t)}+\frac{M_1y}{2\sqrt{\pi t}}e^{-M_1t}\right] \quad (7)$$

where $M_1 = \dfrac{M}{1+m^2}$

The vorticity of flow (7) is given by

$$\zeta = \frac{e^{-(M_1 t+y^2/4t)}}{\sqrt{\pi t}}\left[\frac{k}{4t}\left(1-\frac{2y^2}{t}+\frac{2y^4}{4t^2}\right)+1\right]+\frac{M_1 k}{2\sqrt{\pi t}}e^{-M_1 t} \tag{8}$$

CASE 2 : UNIFORMLY ACCELERATED START

In this case n = 1 introducing the non-dimensional quantities

$$y = y'\left(\frac{U}{\nu^2}\right)^{1/3}, \qquad t = t'\left(\frac{U^2}{\nu}\right)^{1/3},$$

$$u = u'\left(\frac{1}{\nu U}\right)^{1/3} \tag{9}$$

in equation (3) we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - k\frac{\partial^3 u}{\partial y^2 \partial t} - \left(\frac{M}{1+m^2}\right)(u-1) \tag{10}$$

where

$$k = \frac{k'}{\nu}\left(\frac{U^2}{\nu}\right)^{1/3} \quad \text{(Elasticity parameter)}$$

$$M = \frac{\sigma B_o^2}{\rho}\left(\frac{\nu}{U^2}\right)^{1/3} \quad \text{(Magnetic parameter)}$$

The solution of equation (10) using Laplace transform technique is, a

$$u = \frac{1}{2M_1}\left[e^{-M_1 y}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{M_1 t}\right)+e^{M_1 y}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{M_1 t}\right)\right]$$
$$+t+\frac{1}{M_1}\left[e^{-M_1 t}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right)-1\right]-\frac{k}{2\sqrt{\pi t}}e^{-(M_1 t+y^2/4t)} \tag{11}$$

where

$$M_1 = \frac{M}{1+m^2}$$

The vorticity of flow (11) is given by

$$\zeta = \frac{1}{2}\left[e^{M_1 y}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{M_1 t}\right)-e^{-M_1 y}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{M_1 t}\right)\right]+\frac{e^{-y^2/4t}}{M_1\sqrt{t}}\cosh(M_1 t)$$
$$+\frac{k}{2\sqrt{\pi t}}\left(1+\frac{y^2}{2t}e^{-(M_1 t+y^2/4t)}\right) \tag{12}$$

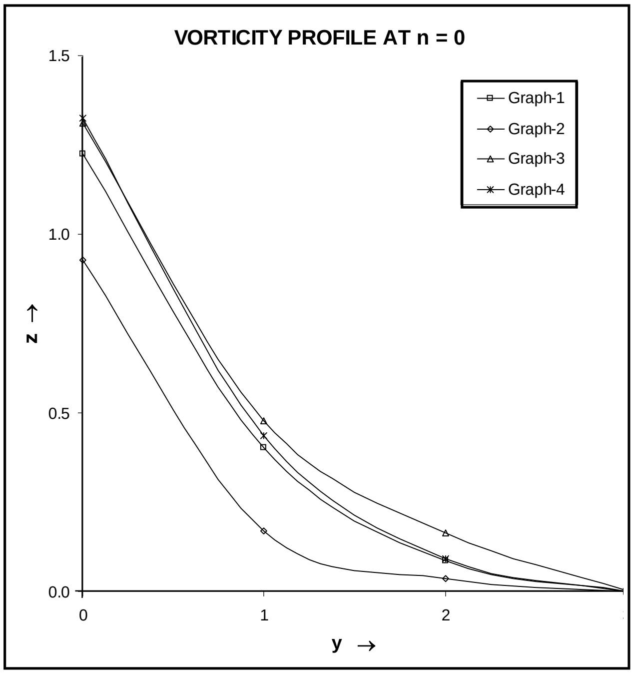
VORTICITY PROFILE AT n = 0
1.5
1.0
0.5
0.0
0
1
2
z →
y →
Graph-1
Graph-2
Graph-3
Graph-4

Fig. 1.

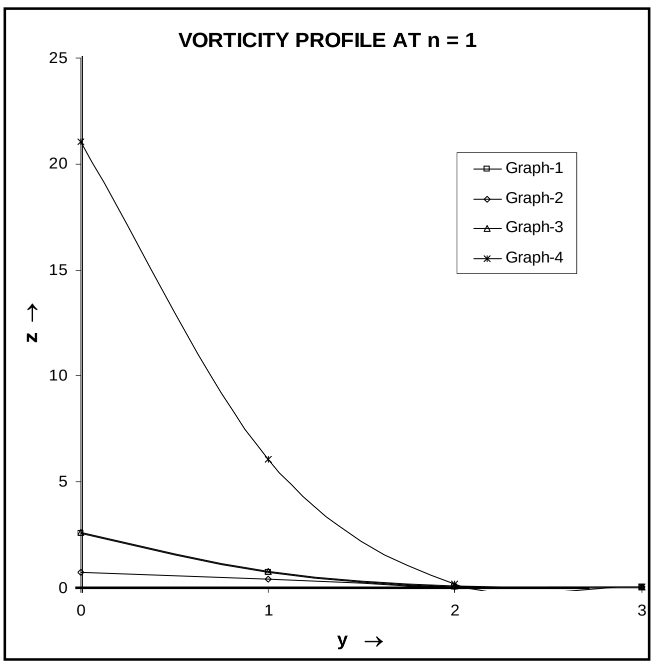
VORTICITY PROFILE AT n = 1
25
20
15
10
5
0
0
1
2
3
z →
y →
Graph-1
Graph-2
Graph-3
Graph-4

Fig. 2.

Table 1. Values of vorticity at n =0 (Impulsive start) for Fig. 1 for t = 0.2 and different values of M, k and m.

y	Graph-1	Graph-2	Graph-3	Graph-4
0	1.225768	0.926849	1.310023	1.326273
1	0.40229	0.168169	0.477531	0.435786
2	0.085174	0.034737	0.162656	0.092926
3	0.000872	0.000355	0.001729	0.000952

Table 2. Values of vorticity at n =1 (Uniformly accelerated start) for Fig. 2 for t = 0.2 and different values of M, k and m.

y	Graph-1	Graph-2	Graph-3	Graph-4
0	2.567298	0.713461	2.598837	21.05916
1	0.735656	0.408809	0.787635	6.049779
2	0.049746	0.042253	0.083208	0.174666
3	0.031579	0.031564	0.063127	0.031821

Result and Discussion

The vorticity profiles at n = 0 & 1 are tabulated in Table 1 & 2 and plotted in Fig. 1 & 2 having four Graphs at t = 0.2 and following different values of M (Magnetic Parameter), k (Elasticity Parameter) and m (Hall currents Parameter).

	M	k	m
For Graph-1	1	0.05	1
For Graph-2	4	0.05	1
For Graph-3	1	0.10	1
For Graph-4	1	0.05	4

It is observed from Fig. 1& 2 that vorticity of the flow is decreasing sharply with the increasing in y. It is also observed that vorticity of the flow in both cases increases with the increase in k and m, but it decreases with the increase in M.

Particular case :

When m is equal to zero, this problem reduces to the problem of Tyagi and Ranjan[3].

Conclusion

The vorticity of the flow increases with the increase in m (Hall currents parameter) in both the cases.

Acknowledgement

The authors are very thankful to the Principal and the Head, Department of Mathematics, D.S. College and S.V. College, Aligarh for providing necessary facilities.

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