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# Two new Fuzzy Models Using Fuzzy Cognitive Maps Model and Kosko Hamming Distance

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## Abstract

In this paper for the first time two new fuzzy models viz Merged Fuzzy Cognitive Maps (MFCMs) models and Specially Merged Linked Fuzzy Cognitive Maps (SMLFCMs) are introduced. To compare the experts opinion a new techniques called Kosko Hamming distance and Kosko Hamming weight are introduced.

*Key words :* Merged graphs, Fuzzy Cognitive Maps, Merged matrices, Merged Fuzzy Cognitive Maps (MFCMs), Specially Merged Linked Fuzzy Cognitive Maps (SMLFCMs), Kosko Hamming Distance

## 1. Introduction

This paper has six sections. The first section is introductory in nature. In the second section the concept of merged graphs and merged matrices are recalled from[11]. For the notion of Fuzzy Cognitive Maps model and its application to social problems please refer[1-11]. In section three the new notion of Merged FCM is introduced and illustrated by an example. Section four introduces the new Specially Merged Linked FCM model. Section five introduces the new technique of comparing the views of the experts using the Kosko Hamming distance and Kosko Hamming weight. The final section gives the conclusion obtained from this research.

## *2 Merged Graphs and their Merged Matrices :*

In this section the definition and properties of merged graphs and their merged matrices are recalled. This is illustrated by examples. For more about merged graphs please refer[11].

*Definition 2.1: Let $G_1 = \{V_1, E_1\}$ and $G_2 = \{V_2, E_2\}$ be two graphs with $V_i$ vertex set and $E_i$ edge set of $G_i$; $i = 1, 2$. We can merge one or more vertices of $G_1$ with $G_2$ and or one or more edges of $G_1$ with $G_2$ which are common to $G_1$ and $G_2$. The resultant graph got so by merging common vertices* *(and or) common edges is a graph defined as the merged graph.*

We will illustrate this definition by an example.

*Example 2.1:* Let $G_1$ be the directed graph given in Figure 2.1.

$G_1$ =

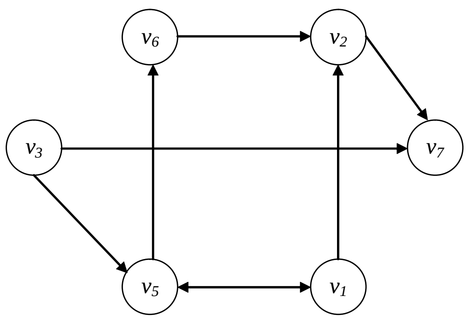


**Figure 2.1**

The matrix related with $G_1$ given in Figure 2.1 be $M_1$, $M_1$ is as follows:

$$M_1 = \begin{array}{cc} & \begin{matrix} v_1 & v_2 & v_3 & v_5 & v_6 & v_7 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_5 \\ v_6 \\ v_7 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $G_2$ be the directed graph which is given in Figure 2.2.

$G_2$ =

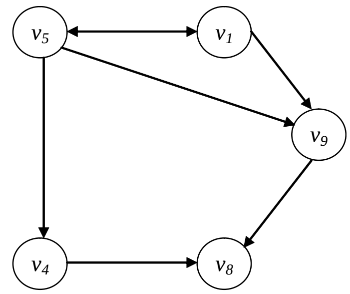


**Figure 2.2**

Let $M_2$ be the matrix associated with the directed graph $G_2$ given in Figure 2.2.

$$M_2 = \begin{array}{cc} & \begin{matrix} v_1 & v_4 & v_5 & v_8 & v_9 \end{matrix} \\ \begin{matrix} v_1 \\ v_4 \\ v_5 \\ v_8 \\ v_9 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Let $G$ be the merged graph of the directed graphs $G_1$ and $G_2$ given in Figure 2.3 got by merging the edge $v_1v_5$ and the vertices $v_1$ and $v_5$. The merged graph $G$ is depicted in Figure 2.3 in the following:

$G$ =

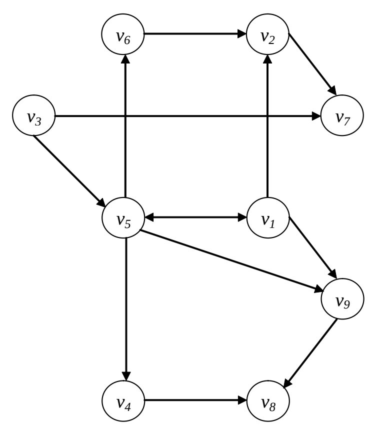


**Figure 2.3**

Let the matrix $M$ denote the merged matrices; $M_1$ and $M_2$ and is the matrix associated with the merged directed graph $G$ which is given in Figure 2.3.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{array} \begin{array}{c} \begin{array}{ccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

This is the way two graphs are merged and the merging is unique and their related matrices are merged and matrix so obtained is defined as the merged matrix.

Next the notion of specially merged linked graph is described by an example.

*Example 2.2:* Let $G_1$, $G_2$ and $G_3$ be the three directed graphs given in the following figures 2.4, 2.5 and 2.6.

$G_1$ =

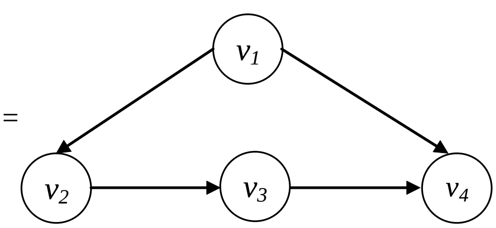


**Figure 2.4**

Let $M_1$ be the matrix associated with the graph $G_1$,

$$M_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $G_2$ be the directed graph given in Figure 2.5.

$G_2$ =

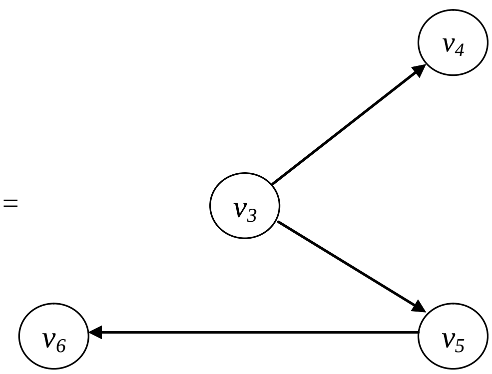


**Figure 2.5**

Let $M_2$ be the matrix associated with the graph $G_2$,

$$M_2 = \begin{array}{c} \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccc} v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $G_3$ be the directed graph given in Figure 2.6.

$G_3$ =

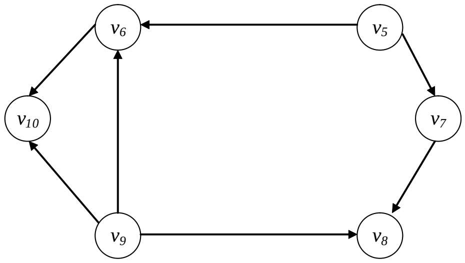


**Figure 2.6**

Let $M_3$ be the matrix associated with the graph $G_3$,

$$M_3 = \begin{array}{c} \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{array} \begin{array}{c} \begin{array}{cccccc} v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We see the graphs $G_1$ and $G_3$ given in Figures 2.4 and 2.6 can not be merged as they do not have a common vertex or edge. However graphs $G_1$ and $G_2$ have $v_3$ to be the common vertex. So $G_1$ and $G_2$ can be merged uniquely only in one way. Likewise graphs $G_2$ and $G_3$ have $v_5$ and $v_6$ as common vertices so they can be merged in a unique way. Thus all the three graphs $G_1$, $G_2$ and $G_3$ can be merged in a unique way into a single graph $G$ given in Figure 2.7.

$G$ =

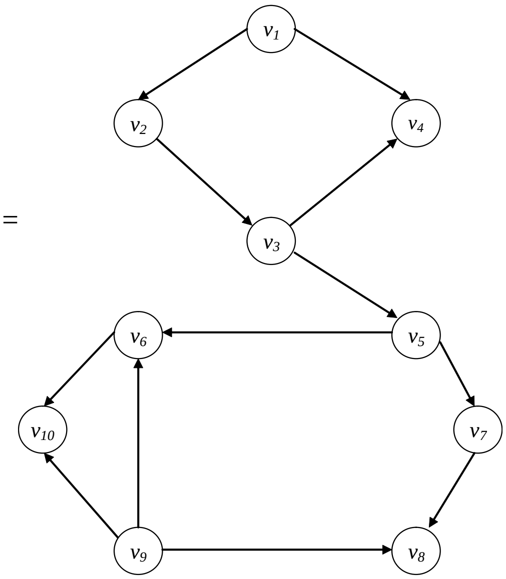


**Figure 2.7**

Let $M$ denote the specially linked merged matrix of the matrices $M_1$, $M_2$ and $M_3$ which is also the matrix associated with the merged linked graph $G$.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{array} \begin{array}{c} \begin{array}{cccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

This sort of merging more than two graphs under certain constraints is known as the specially merged linked graphs.

In the next section how these concepts are used in the construction of merged FCMs is described.

*3 Definition of New Merged FCMs and their properties :*

In this section the notion of Merged Fuzzy Cognitive Maps (MFCMs) is introduced. Merged FCMs are built using the concept of merged graphs and the related merged matrices and described by an example.

*Definition 3.1: Let* $C = \{C_1, \ldots, C_n\}$ *be the n nodes associated with a real world problem. Suppose t experts want to work with this problem using FCMs but only using some selected nodes from the set of nodes C.*

*Let the directed graphs given by the t experts be* $G_1, G_2, \ldots, G_t$ *such that the vertex set of the graph* $G_i$ *with* $G_j$ *is non empty for* $i \neq j$; $G_i \cap G_j \neq \phi$; $1 \quad i, j \leq t$. *Then we can merge some graphs say k of them,* $k \leq t$ *from* $G_1, \ldots, G_t$ *so that the vertices of all these graphs give all the nodes of the set C.*

*Let G be the merged graph and the FCMs associated with G will be known as the Merged Fuzzy Cognitive Maps (MFCMs) and the connection matrix associated with G will be known as the merged connection matrix of the MFCMs or the merged dynamical system of the FCMs.*

We will first illustrate this situation by an example.

*Example 3.1:* Let us suppose we have $C = \{C_1, C_2, C_3, \ldots, C_{12}\}$ to be the set of nodes/attributes associated with a problem. Let five experts work with the problem using FCMs and the nodes from the set $C$.

Suppose the first expert wants to work with the set of nodes given by $X_1$ where $X_1 = \{C_1, C_2, C_3, C_4, C_8\} \subseteq C$ and the second expert wishes to work with the set of nodes given by $X_2$, where $X_2 = \{C_3, C_7, C_5, C_{12}\} \subseteq C$.

The third expert works with the set of nodes given by the subset $X_3$, where $X_3 = \{C_1, C_7, C_{10}, C_{11}\} \subseteq C$. Let the forth expert work with the nodes $C_6, C_9, C_1, C_{10}, C_{12}$ given by the set $X_4 = \{C_6, C_9, C_1, C_{10}, C_{12}\} \subseteq C$.

The fifth expert works with $X_5 = \{C_6, C_5, C_{10}, C_2, C_9, C_7\} \subseteq C$. Now we can get the merged FCMs in two ways. Taking the nodes $X_1 \cup X_2 \cup X_3 \cup X_4$ so that we get the merged graph $G$ of the graphs $G_1, G_2, G_3$ and $G_4$ or $X_1 \cup X_2 \cup X_3 \cup X_5$ that is we get the merged graph $G'$ of the graphs $G_1, G_2, G_3$ and $G_5$.

So we get two integrated merged FCMs model to work with. Let us consider the directed graphs given by the five experts.

Let $G_1$ be the directed graph given in Figure 3.1 by the first expert using the set of attributes $X_1$.

$G_1$ =

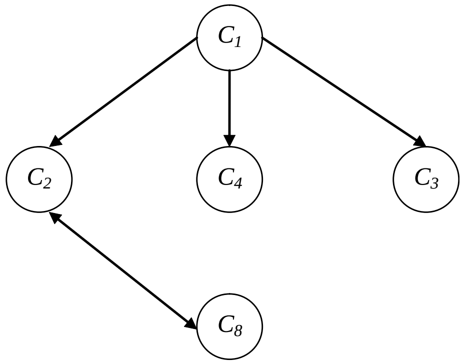


**Figure 3.1**

Let $M_1$ be the matrix associated with the graph $G_1$.

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_8 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $G_2$ be the directed graph given by the second expert using the attributes $X_2 = \{C_3, C_7, C_5, C_{12}\}$ given in Figure 3.2.

$G_2$ =

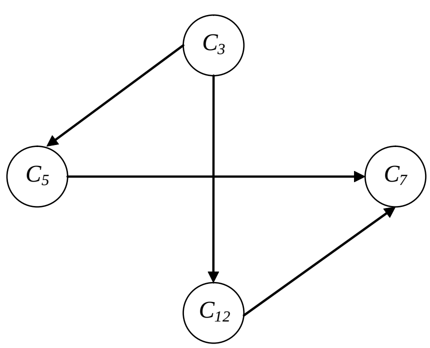


**Figure 3.2**

Let $M_2$ be the matrix associated with the graph $G_2$.

$$M_2 = \begin{array}{c} \\ C_3 \\ C_5 \\ C_7 \\ C_{12} \end{array} \begin{array}{c} \begin{array}{cccc} C_3 & C_5 & C_7 & C_{12} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Let $G_3$ be the directed graph given in Figure 3.3 given by the third expert using the nodes $X_3 = \{C_1, C_7, C_{10}, C_{11}\};$

$G_3$ =

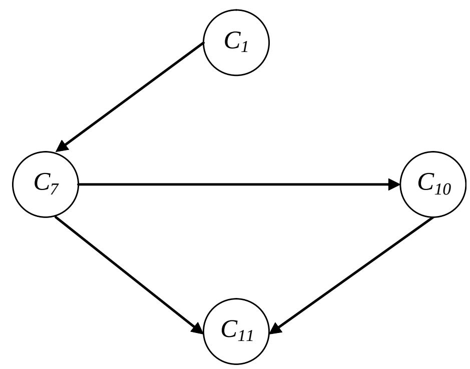


**Figure 3.3**

Let $M_3$ be the matrix associated with the graph $G_3$.

$$M_3 = \begin{array}{c} \\ C_1 \\ C_7 \\ C_{10} \\ C_{11} \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_7 & C_{10} & C_{11} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $G_4$ be the directed graph provided by the fourth expert using the set of nodes $X_4 = \{C_6, C_9, C_1, C_{12}, C_{10}\}$ given in Figure 3.4.

$G_4$ =

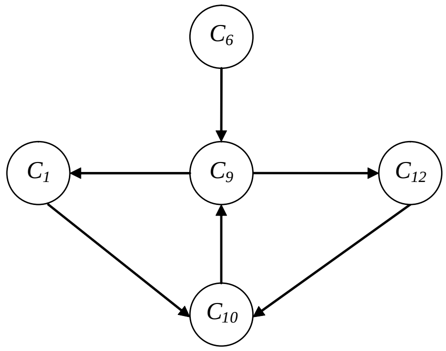


**Figure 3.4**

Let $M_4$ be the matrix associated with the graph $G_4$.

$$M_4 = \begin{array}{c} \\ C_1 \\ C_6 \\ C_9 \\ C_{10} \\ C_{12} \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_6 & C_9 & C_{10} & C_{12} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Using $X_5 = \{C_6, C_5, C_{10}, C_2, C_9, C_7\}$, let $G_5$ be the directed associated with the fifth expert given in Figure 3.5.

$G_5$ =

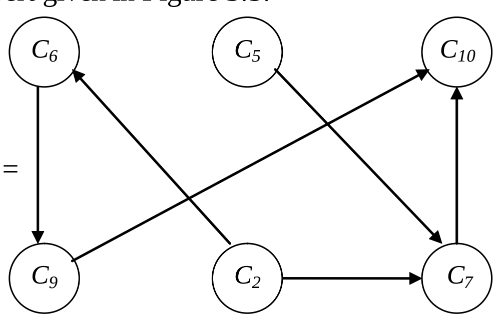


**Figure 3.5**

Let $M_5$ be the matrix associated with the graph $G_5$.

$$M_5 = \begin{array}{c} \\ C_2 \\ C_5 \\ C_6 \\ C_7 \\ C_9 \\ C_{10} \end{array} \begin{array}{c} \begin{array}{cccccc} C_2 & C_5 & C_6 & C_7 & C_9 & C_{10} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let $M_1, M_2, \ldots, M_5$ be the matrices associated with the graphs $G_1, G_2, \ldots, G_5$ respectively. Let the merged matrix of the matrices be $M$ which is also the merged connection matrix $M$ of the specially linked merged graph $G$ given in Figure 3.6.

Now to get the integrated merged FCMs we have to merge the graphs $G_1, G_2$, $G_3$ and $G_4$ or $G_1$, $G_2$, $G_3$ and $G_5$. So we get in total using merged FCMs two integrated merged FCMs using all the 12 attributes. The merged graph $G$ of the experts 1 to 4 is given in Figure 3.6.

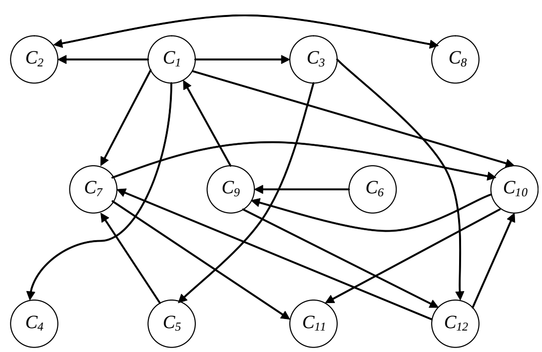


**Figure 3.6**

Using this merged connection matrices $M_1$, $M_2$, $M_3$ and $M_4$ of graphs $G_1$, $G_2$, $G_3$ and $G_4$ respectively we can study the integrated merged dynamical system of the integrated MFCMs.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now using the experts 1, 2, 3, and 5 we get the merged graph $G'$ of the four directed graphs of the FCM given in Figure 3.7.

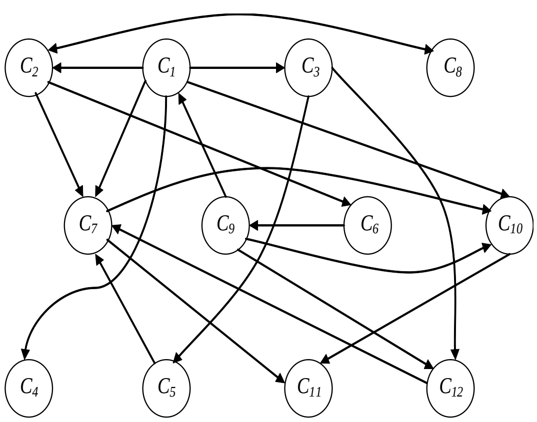


**Figure 3.7**

Let the merged connection matrix of the matrices $M_1$, $M_2$, $M_3$ and $M_5$ of the merged graph $G'$ of Figure 3.3.7 be $M'$ which is as follows:

$$M' = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \\ C_{12} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} & C_{12} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Thus we can find the merged connection matrix $M'$ of the FCM and study the problem. The advantages of using this new merged FCMs models are;

1. Experts can choose any number of attributes from the given set of attributes so that they can be free to do work with the problem with their choice.
2. When the number of attributes is a small number; working is easy and apt.
3. While getting merged model by combining all the experts opinion (so no expert is left out), everyone is given the same degree of importance.
4. By combining them differently using different sets of experts we get several merged FCMs for the same problem.
5. The values in the connection merged

integrated matrices need not be thresholded for they take only values from the set $\{-1, 1, 0\}$.

*4 Definition and Description of Specially Merged Linked FCMs :*

The Specially Merged Linked Fuzzy Cognitive Maps (SMLFCMs) model are defined and described in the following.

*Definition 4.1: Let $C = \{C_1, \ldots, C_n\}$ be the n attributes of a problem with which t experts $E_1, E_2, \ldots, E_t$ work using FCMs model. Each of the experts work with $X_i$ set of attributes from the n attribute set C; that is $X_i \subseteq C$, $X_i$ the subset of C. Now the sets $X_i$'s $1 \leq i \leq t$ are so formed such that $X_i \cap X_{i+1} \neq \phi$, $X_i \cap X_j = \phi$, for $j = 3, 4, \ldots t$, $i = 1, 2, \ldots, t - 1$. That is $X_1 \cap X_2 \neq \phi$, $X_2 \cap X_3 \neq \phi$ but $X_2 \cap X_i = \phi$, for $i = 4, 5, 6, \ldots, t$; similarly is $X_3 \cap X_4 \neq \phi$ but $X_3 \cap X_i = \phi$, for $i = 5, 6, \ldots, t$ and so on.*

*Thus we see even when $G_1, G_2, \ldots, G_t$ are the directed graphs given by the experts $E_1, E_2, \ldots, E_t$ using the set of nodes $X_1, X_2, \ldots, X_t$ respectively we see we cannot merge any $G_i$ with $G_j$. $G_i$ can be merged with $G_j$ provided $j = i + 1$ or $i - 1$ ($i \neq 1$). Thus we defined this special type of merging as specially merged linked graphs. Let $M_1, M_2, \ldots, M_n$ be the n matrices associated with the directed graphs $G_1, G_2, \ldots, G_n$ respectively. Let M be the specially merged linked matrix associated with the specially merged linked graph.*

*We call the FCMs associated with these specially merged linked graphs are defined as Specially Merged Linked Fuzzy Cognitive Maps (SMLFCMs) model.*

We will illustrate this situation by an example.

*Example 4.1:* Let $C = \{C_1, C_2, \ldots, C_{11}\}$ be the collection of nodes associated with a problem. Let three experts $E_1$, $E_2$ and $E_3$ work with the same problem using the FCMs model choosing a few nodes from $C$. Let the expert $E_1$ choose to work with the nodes $X_1 = \{C_1, C_2, C_3, C_7, C_8\} \subseteq C$.

Let the expert $E_2$ work with the nodes $X_2 = \{C_1, C_6, C_7, C_4, C_9\} \subseteq C$ and the expert $E_3$ chooses to work with the nodes $X_3 = \{C_5, C_6, C_{10}, C_{11}, C_4\}$.

Let $G_1$, $G_2$ and $G_3$ be the directed graphs associated with the experts $E_1$, $E_2$ and $E_3$ respectively given in Figures 4.1, 4.2 and 4.3.

$G_1$ =

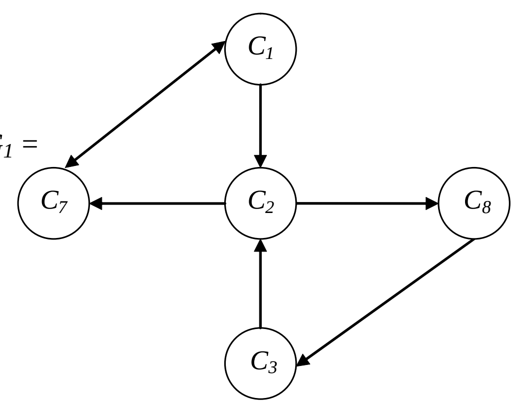


**Figure 4.1**

is the directed graph given in Figure 4.1 of the FCMs given by the first expert.

$G_2$ =

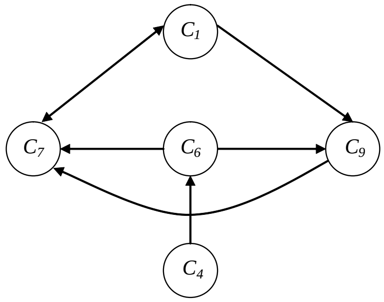


**Figure 4.2**

Let $G_2$ be the directed graph of the FCMs given by the second expert as given in Figure 4.2.

Let $G_3$ be the directed graph given by the third expert which is given in the following Figure 4.3:

$G_3$ =

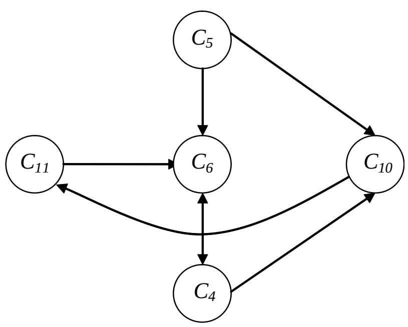


**Figure 4.3**

Let

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_7 \\ C_8 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_7 & C_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}, \quad M_2 = \begin{array}{c} \\ C_1 \\ C_4 \\ C_6 \\ C_7 \\ C_9 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} C_1 & C_4 & C_6 & C_7 & C_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

and

$$M_3 = \begin{array}{c} \\ C_4 \\ C_5 \\ C_6 \\ C_{10} \\ C_{11} \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} C_4 & C_5 & C_6 & C_{10} & C_{11} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

be the connection matrices associated with the directed graphs $G_1$, $G_2$ and $G_3$ respectively given in Figures 4.1, 4.2 and 4.3.

Now we see we cannot merge $G_1$ and $G_3$ as they do not have a common node or an edge. Now we can merge only in one way $G_1$ with $G_2$ and $G_2$ with $G_3$ which is the specially linked merged graph $G$ given in the following Figure 4.4.

$G$ =

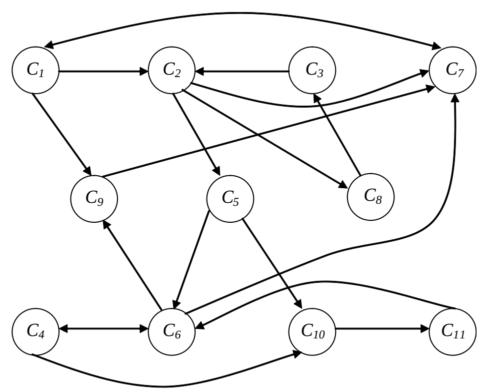


**Figure 4.4**

Let $M_I$ denote the specially linked merged matrix of the matrices $M_1$, $M_2$ and $M_3$ of the directed graphs $G_1$, $G_2$ and $G_3$. Using $G$ we can find the integrated merged linked special connection matrix $M$ of the specially merged linked matrices of the matrices $M_1$, $M_2$ and $M_3$ which is also the connection matrix of the specially linked graph $G$ of the dynamical system. That matrix will serve as the dynamical system of the specially merged linked FCMs model given in the following.

The matrix $M$ of $G$ is got by merging $M_1$, $M_2$ and $M_3$ is as follows:

$$M_1 = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \\ C_{10} \\ C_{11} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 & C_{10} & C_{11} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} .$$

Next we proceed on to define Kosko Hamming distance function in the next section.

## *5 Kosko Hamming Distance Function and Kosko Hamming Weight :*

We know Hamming distance measures the number of coordinates a row vector $1 \times n$ differs from another $1 \times n$ row vector. So in the vector space $V^n = \{(a_1, \ldots, a_n) \mid a_i \in F; 1 \leq i \leq n\}$ defined over the field $F$ we can define the Hamming distance for any $x, y \in V^n$ as $d(x, y)$ = number of coordinates in which $x$ and $y$ differ.

However in case of Kosko Hamming distance function we can define the distance function $d_k$ only if these conditions are satisfied.

1. We need to have basically a fuzzy model say a FCMs model.
2. We need atleast two experts working with the same number of concepts/nodes using only the same type of fuzzy model with same set of attributes on the same problem say for instance a FCM model.
3. We can define the Kosko Hamming distance function $d_k$ on the resultant state vectors given by two experts that is on the hidden pattern of a same initial state vector $X_i$ used by both the experts using FCMs model.

So the concept of Kosko Hamming distance cannot be defined for any resultant state vectors but only those resultant state vectors whose initial input state vector is the same. Thus Kosko Hamming distance is distinctly different from the Hamming distance. This $d_k$ also measures only the number of places in which the resultant state vectors given by two experts for the same initial state vector differs.

Now we proceed onto define the notion of Kosko Hamming distance.

*Definition 5.1: Let some $t$ experts $E_1, E_2, \ldots, E_t$ work with the same problem using the same set of attributes or nodes $C = \{C_1, C_2, \ldots, C_n\}$ using FCMs model.*

*Let $M_1, M_2, \ldots, M_t$ be the $t$ $(n \times n)$ connection matrices of the FCMs associated* *with the experts $E_1$, $E_2$, …, $E_t$ respectively. Suppose X be the initial state vector for which the hidden patterns using the dynamical system $M_1$, $M_2$, …, $M_t$ is given by $Y_1$, $Y_2$, …, $Y_t$ respectively. Now the Kosko Hamming distance function $d_k$ is defined as $d_k(Y_i, Y_j)$ = number of coordinates in which $Y_i$ is different from $Y_j$; $i \neq j$; $i \leq i$, $j \leq t$; $0 \leq d_k(Y_i, Y_j) \leq n - 1$.*

The use of this function is that it measures the closeness or the non closeness of the resultant vectors of two experts for the same initial state vector *X* of the problem.

So if the deviation, that is the value of $d_k(Y_i, Y_j)$ is very high we can analyse the experts $E_i$ and $E_j$ separately for that initial point *X* as well as for the other initial points also. If the functional value $d_k(Y_i, Y_j)$ is small we accept that both the experts hold the same view for the particular initial state vector *X*.

*Definition 5.2: Let the experts, nodes and connection matrices be as in definition 5.1.*

*Let the initial state vector be X, $Y_1$, …, $Y_t$ be the hidden patterns given by the dynamical systems $M_1$, $M_2$, …, $M_t$ respectively. The Kosko Hamming weight function is defined and denoted by $w_k(Y_i) = d_k(Y_i, X)$; number of coordinates in which $Y_i$ differs from X; $i = 1, …, t$.*

This new notion of Kosko Hamming weight function helps one in finding the impact of the particular node/nodes in *X* which are taken in the on state over the dynamical system that is over the other nodes. If $w_k(Y_i)$ is very large it implies that the node has a very high impact on the system that is on other nodes which it has made to on state by its on state.

If $w_k(Y_i)$ is small it implies the node/ nodes are taken in the on state has no impact on the other nodes or equivalently no impact over the system.

## 6. Conclusions

In this chapter we have introduced two new mathematical tools to study and analyse the FCMs model, the relation between the experts opinion and the impact on one node over other nodes in that dynamical system. Both the models can give the integrated view of all the experts. This model thus saves time. This model satisfies all experts because all are given equal status about the problem[12-13].

The first tool helps in replacing the combined FCMs and makes the working easy by considering less number of attributes by getting the integrated merged FCMs model or special integrated linked merged FCMs model. The former can give more than one FCMs model so that comparison can be made, however the latter technique can give only one integrated merged FCMs model. Finally the tool of Kosko Hamming distance function and Kosko Hamming weight function can study the influence of a node over other nodes and the comparison of experts view on the problem.

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