Ultra Scientist Vol. 24(1)B, 7-18 (2012).

Mixed Convection Flow from Vertical Plate Embedded in Non-Newtonian Fluid Saturated Non- Darcy Porous Medium with Melting Effect

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(Acceptance Date 29th November, 2011)

Abstract

We analyzed in this paper the problem of mixed convection along a vertical plate in a non-Newtonian fluid saturated non-Darcy porous medium in the presence of melting and thermal dispersion-radiation effects for aiding and opposing external flows. Similarity solution for the governing equations is obtained for the flow equations in steady state. The equations are numerically solved by using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D), radiation (R), temperature ratio (Cr), inertia (F), mixed convection (Ra/Pe) and Nusselt number on velocity distribution and temperature are examined for aiding and opposing external flows.

Key words: Porous medium, Non-Newtonian Fluid, Melting, Thermal Dispersion, Radiation.

Introduction

Convection heat transfer in porous media in the presence of melting effect has received some attention in recent years. This stems from the fact that this topic has significant direct application in permafrost melting, frozen ground thawing, casting and bending processes as well as phase change metal. The study of melting effect is considered by many researchers in Newtonian fluids. Non-Newtonian power law fluids are so wide spread in industrial process and in the environment. The melting phenomena on free convection from a vertical front in a non- Newtonian fluid saturated porous matrix are analyzed by Poulikakos and Spatz[1]. Nakayama and Koyama[2] studied the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulikakos[3] examined the problem of double diffusive convection from a vertical plate in a porous medium saturated with a non-Newtonian power law fluid. Considering geothermal and oil reservoir engineering applications, Nakayama and Shenoy[4] studied a unified similarity transformation for Darcy and non-Darcy forced, free and mixed convection heat transfer in non-Newtonian inelastic fluid saturated porous media.

Effect of melting and thermo-diffusion on natural convection heat mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium was studied by R.R. Kairi and P.V.S.N.-Murthy[5]. It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. The non-linear behavior of non-Newtonian fluids in a porous matrix is quite different from that of Newtonian fluids in porous media. The prediction of heat or mass transfer characteristics for mixed or natural convection of non-Newtonian fluids in porous media is very important due to its practical engineering applications such as oil recovery and food processing. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion. On the other hand it is worth mentioning that heat transfer simultaneous radiation and convection is very important in the context of space technology and processes involving high temperatures.

Recently, A.Y. Bakier *et al.*[6] studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. He developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface. More recently Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Ali. J. Chamka *et.al.*[7]. They obtained representative flow and heat transfer results for various combinations of physical parameters.

The present paper is aimed at analyzing the effect of melting and thermal dispersion-radiation on steady mixed convective heat transfer from a vertical plate embedded in a non-Newtonian power law fluid saturated non-Darcy porous medium for aiding and opposing external flows.

Mathematical formulation :

A mixed convective heat transfer in a non-Darcy porous medium saturated with a homogeneous non-Newtonian fluid adjacent to a vertical plate, with a uniform wall temperature is considered. This plate constitutes the interface between the liquid phase and the solid phase during melting inside the porous matrix at steady state. The plate is at a constant temperature T_m at which the material of the porous matrix melts. Fig.1 shows the coordinate and the flow model. The x-coordinate is measured along the plate and the y-coordinate normal to it. The solid phase is at temperature $T_0 < T_m$. A thin boundary layer exists close to the right of vertical plate and temperature changes smoothly through this layer from T_m to T_∞ ($T_m < T_\infty$) which is the temperature of the fluid phase.

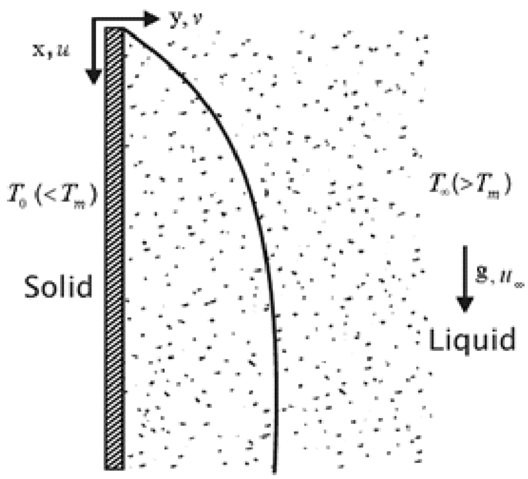


Fig. 1. (Schematic diagram of the problem)

Taking into account the effect of thermal dispersion the governing equations for steady non-Darcy flow in a non-Newtonian fluid saturated porous medium can be written as follows.

The continuity equation is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

The momentum equation is

$$\frac{\partial u^n}{\partial y} + \frac{C\sqrt{K}}{\nu}\frac{\partial u^2}{\partial y} = \mp \frac{Kg\beta}{\nu}\frac{\partial T}{\partial y} \tag{2}$$

The energy equation is

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right) - \frac{1}{\rho C_p}\frac{\partial q}{\partial y} \tag{3}$$

In the above equations, the term which represents the buoyancy forced effect on the flow field has ± signs. The plus sign indicates aiding buoyancy flow where as the negative sign stands for buoyancy opposed flow. Here u and v are the velocities along x and y directions respectively, n is the power-law fluid viscosity index, T is Temperature in the thermal boundary layer, K is Permeability, C is Forchheimer empirical constant, β is coefficient of thermal expansion, ν is Kinematics viscosity, ρ is Density, Cp is Specific heat at constant pressure, g is acceleration due to gravity, and thermal diffusivity $\alpha=\alpha_m+\alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb[8], the dispersion thermal diffusivity α_d is proportional to the velocity component *i.e* $\alpha_d=\gamma ud$, where γ is the dispersion coefficient and d is the mean particle diameter. The radiative heat flux term q is written using the Rosseland approximation (Sparrow and Cess[9], Raptis[10]) as

$$q = -\frac{4\sigma_R}{3a}\frac{\partial T^4}{\partial y} \tag{4}$$

Where σ_R is the Stefan - Boltzmann constant and 'a' is the mean absorption coefficient.

The physical boundary conditions for the present problem are

$$y=0, T=T_m,\ k\frac{\partial T}{\partial y} = \rho[h_{sf} + C_s(T_m - T_0)]v \tag{5}$$

$$\text{and}\quad y\to\infty,\ T\to T_\infty,\ u=u_\infty \tag{6}$$

Where h_{sf} and C_s are latent heat of the solid and specific heat of the solid phases respectively and u_∞ is the assisting external flow velocity, $k=\alpha\rho C_p$ is the effective thermal conductivity of the porous medium. The boundary condition (5) means that the temperature on the plate is constant and thermal flux of heat conduction to the melting surface is equal to the sum of the heat of melting and the heat required for raising the temperature of solid to its melting temperature T_m.

Introducing the stream function ψ with

$$u = \frac{\partial\psi}{\partial y}, \text{and } v = -\frac{\partial\psi}{\partial x}$$

The continuity equation (1) will be satisfied and the equations (2) and (3) transform to

$$\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^n + \frac{C\sqrt{K}}{\nu}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^2 = \mp\frac{Kg\beta}{\nu}\frac{\partial T}{\partial y} \quad (7)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[(\alpha_m + \gamma\frac{\partial\psi}{\partial y}d)\frac{\partial T}{\partial y}\right] + \frac{4\sigma_R}{3\rho C_p a}\frac{\partial}{\partial y}\left[\frac{\partial T^4}{\partial y}\right] \quad (8)$$

Introducing the similarity variables as $\psi = f(\eta)(\alpha_m u_\infty x)^{1/2}$, $\eta = \left(\frac{u_\infty x}{\alpha_m}\right)^{\frac{1}{2}}\left(\frac{y}{x}\right)$, $\theta(\eta) = \frac{T - T_m}{T_\infty - T_m}$,

The momentum equation (7) and energy equation (8) are reduced to

$$nf^{11}f^{1^{n-1}} + 2Ff^1f^{11} = \mp\left(\frac{Ra_x}{Pe_x}\right)^n\theta^1 \quad (9)$$

$$(1 + Df^1)\theta^{11} + \left(\frac{1}{2}f + Df^{11}\right)\theta^1 + \frac{4}{3}R\left[\left((\theta + C_r)^3\theta^{11} + 3\theta^{1^2}(\theta + C_r)^2\right)\right] = 0 \quad (10)$$

Where the prime symbol denotes the differentiation with respect to the similarity variable η and R_{a_x}/P_{e_x} is the mixed convection parameter, $Ra_x = \frac{x}{\alpha}\left(\frac{g\beta k(T_\infty - T_m)}{\nu}\right)^{\frac{1}{n}}$ is the local Rayleigh number, $Pe_x = \frac{u_\infty x}{\alpha}$ is the local peclet number. $F = f_0(Pe_d)^{2-n}$, $f_0 = \left(\frac{\alpha}{d}\right)^{2-n}\left(\frac{C\sqrt{K}}{\nu}\right)$ is the non-Darcian parameter. P_{e_d} is the pore diameter dependent peclet number. $D = \frac{\gamma d u_\infty}{\alpha_m}$ is the dispersion parameter. $C_r = \frac{T_m}{T_\infty - T_m}$ is the temperature ratio, $R = \frac{4\sigma_R(T_\infty - T_m)^3}{ka}$ ka is the radiation parameter.

Taking into consideration, the thermal dispersion effect together with melting, the boundary conditions (5) and (6) take the form

$\eta=0, \theta=0, f(0)+\{1+Df^1(0)\}2M\theta^1(0)=0.$ (11)

And $\eta\to\infty$, $\theta=1$, $f^1=1$. (12)

Where $M = \frac{C_f(T_\infty - T_m)}{h_{sf} + C_s(T_m - T_0)}$ is the melting parameter. The local heat transfer rate from the surface of the plane is given by $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$. The Nusselt number is $N_u = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)}$, where h is the local heat transfer coefficient and k is the effective thermal conductivity of the porous medium, which is the sum of the molecular thermal conductivity k_m and the dispersion thermal conductivity k_d. The modified Nusselt number is obtained as

$$\frac{Nu_x}{(Pe_x)^{\frac{1}{2}}} = \left[1 + \frac{4}{3}R(\theta(0) + C_r)^3 + Df^1(0)\right]\theta^1(0) \quad (13)$$

Solution Procedure :

The dimensionless equations eq. (9) and eq.(10) together with the boundary conditions (11) and (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. The solution thus obtained is matched with the given values of f^1 (∞) and θ (0). In addition the boundary condition $\eta \rightarrow \infty$ is approximated by η_{max} =8 which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. Numerical computations are carried out for F=0, 0.5, 1; D=0, 0.5, 1; Ra/Pe=1; M=0, 0.4,0.8,1.2,1.6,2; n=0.5, R=0,0.5,1; Cr=0.1, 0.5,1.

Results and Discussion

In order to get clear insight on the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The parameters governing the physics of the present study are the melting (M), the mixed convection (Ra/Pe), the inertia (F), thermal dispersion (D), and Fluid viscosity index (n), radiation(R), temperature ratio (Cr). The numerical computations were carried out for the fixed value of buoyancy parameter Ra/Pe =1 for both the aiding and opposing external flows. The results of the parametric study are shown in figures 2-25. Fig. 2 shows the effect of melting parameter M on velocity profiles for both aiding and opposing external flows. It is observed that different behavior exists between the velocity profiles in the presence of solid phase melting effect in the case of aiding and opposing external flows. The increase in the melting parameter M causes the increase of the velocity profiles for the case of aiding flow. Whereas the opposite behavior for the velocity profiles as M increases is found in the case of opposing flow case.

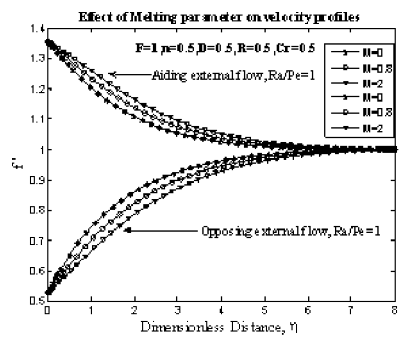


Fig. 2

Fig. 3 and Fig. 4 show the effect of the melting parameter M on temperature distributions for aiding and opposing external flows respectively. It is observed that as increasing the value of the melting parameter M, the temperature distributions decrease for both cases of aiding and opposing external flows.

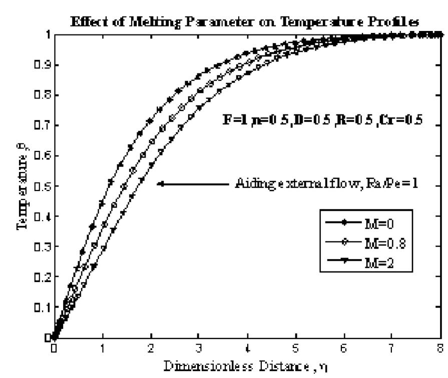
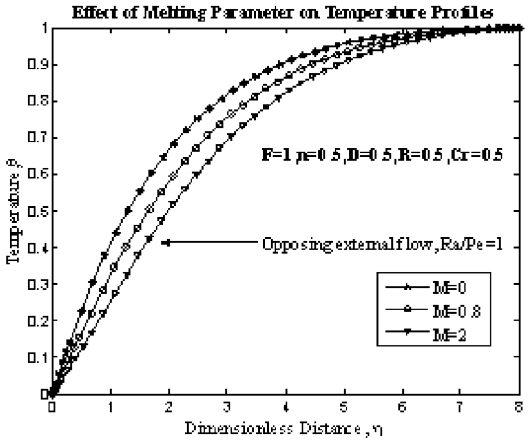


Fig. 3



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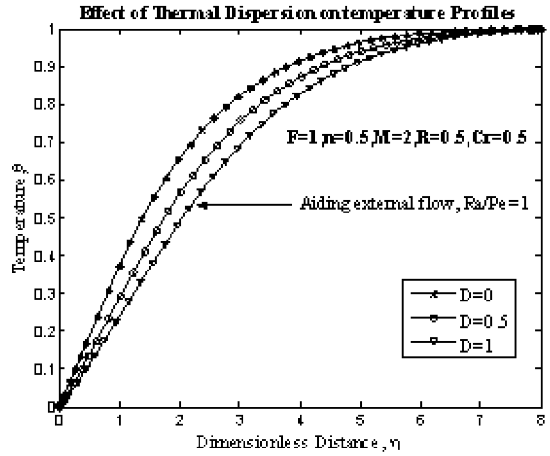

Fig. 6

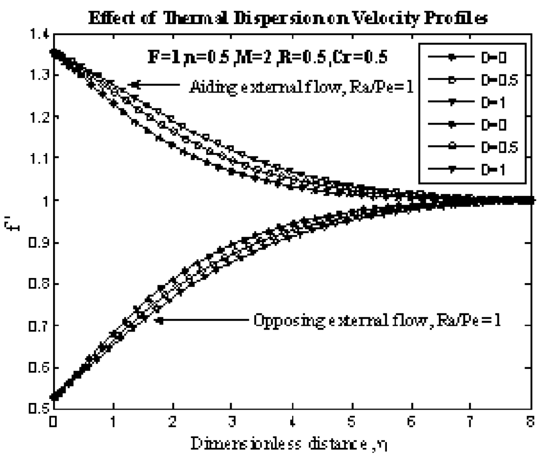

Fig. 5

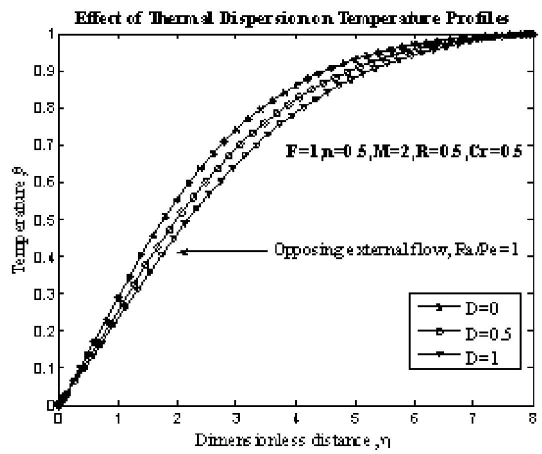

Fig. 7

The effects of thermal dispersion Parameter D on the velocity profiles for both cases of aiding and opposing external flow conditions are plotted in Fig. 5. It is noted that for the case of aiding flow the velocity profiles increase with increase in the value of D. But this effect is found opposite in the case of opposing flow.

The effects of thermal dispersion parameter D on temperature distributions for aiding and external flow conditions are plotted in Fig. 6 and Fig. 7 respectively. It is noted that increasing the values of D leads to decrease in the liquid temperature distributions in both cases.

Fig. 8 shows the effects of power law

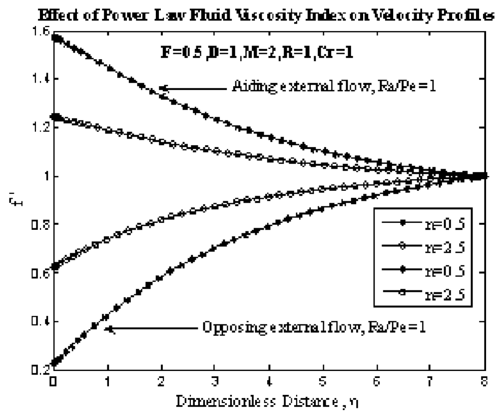

Fig. 8

fluid viscosity index n on velocity profiles for both aiding and opposing flow cases. It is observed that in aiding flow case as n value increases, velocity profiles decreases. Whereas opposite behavior obtained in velocity profiles in opposing flow case.

Fig. 9 and Fig. 10 show the effects of power law fluid viscosity index n on temperature distributions for both aiding and opposing flow cases respectively. Significant effect is not found. In aiding flow case as n increases, the temperature distributions decrease. Whereas opposite result is found in opposing flow case.

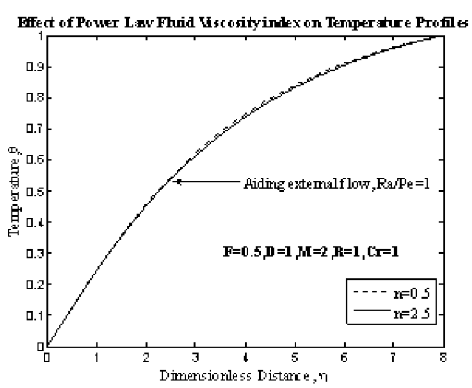

Fig. 9

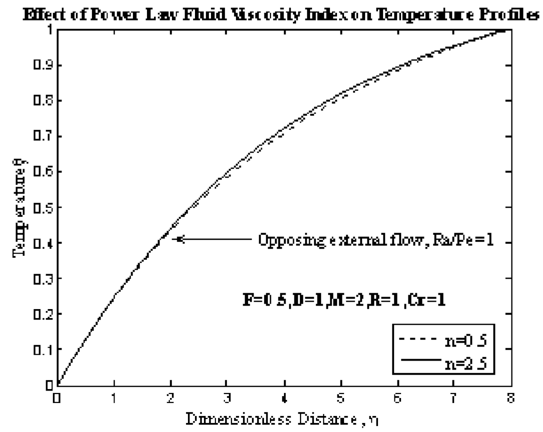

Fig. 10

Fig. 11 shows the effect of radiation parameter R on velocity profiles for both aiding and opposing external flows. The increase in the radiation parameter R causes the increase of the velocity profiles for aiding flow case. Whereas in the case of opposing flow the result is found opposite.

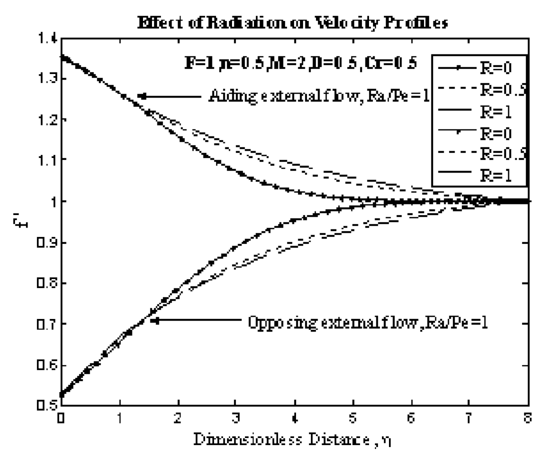

Fig. 11

Fig. 12 and Fig. 13 show the effect of radiation parameter R on temperature distributions for aiding and opposing external flow conditions respectively. It is observed that same phenomena exist in both cases. As R increases the temperature distributions decrease for both cases.

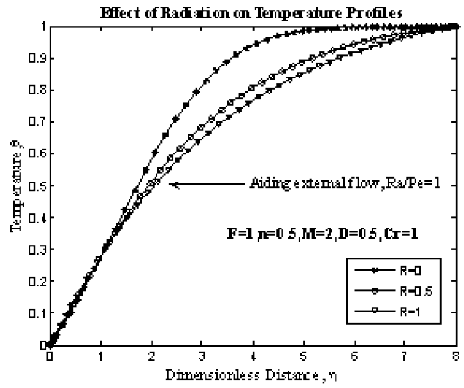

Fig. 12

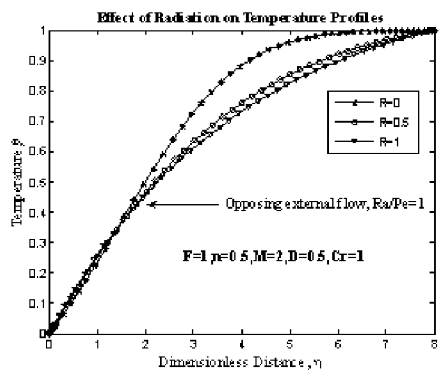

Fig. 13

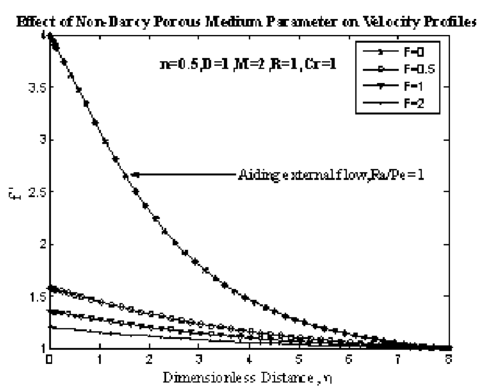

Fig. 14

Fig. 14 and Fig. 15 show the effect of non-Darcy porous medium parameter F on velocity profiles for aiding and external flow conditions respectively. In the aiding flow case the velocity profiles decrease with increasing the value of F. But in the opposing flow case the velocity profiles increase with the increase of F value.

Fig. 16 and Fig. 17 show the effects of non-Darcy porous medium parameter F on temperature distributions for aiding and opposing flow cases respectively. In aiding flow

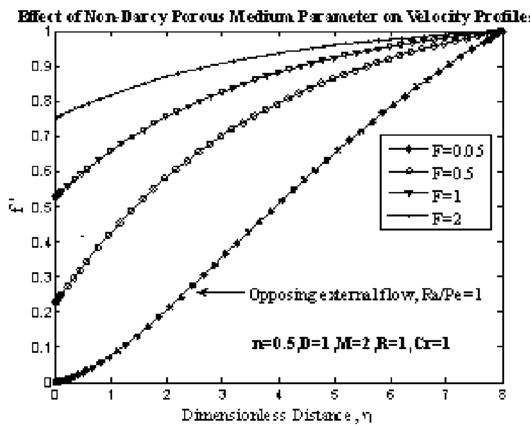

Fig. 15

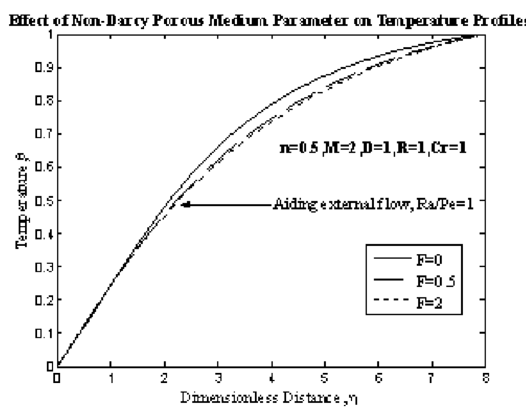

Fig. 16

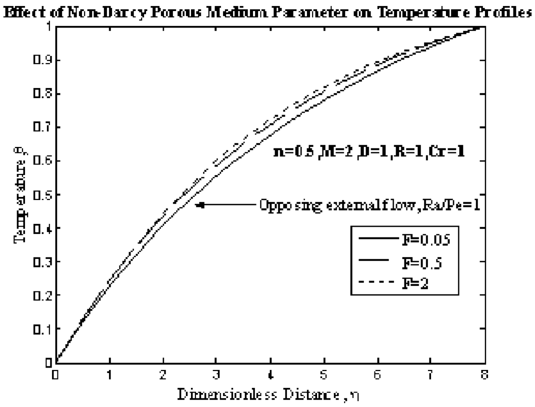

Fig. 17

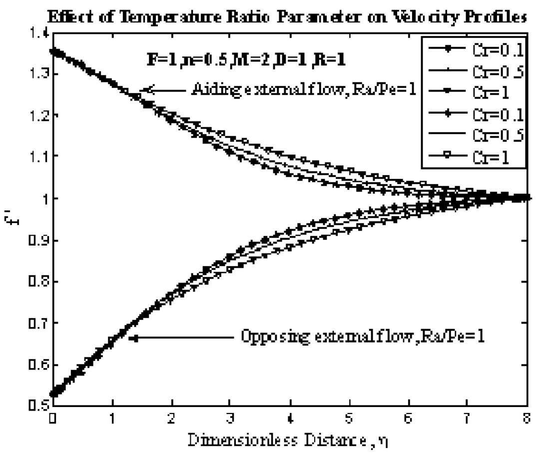

Fig. 18

case the temperature distributions decrease with the increase of F value. Whereas in opposing flow case the temperature distributions increase with the increase of F value.

The effects of temperature ratio parameter Cr on velocity profiles for aiding and opposing external flow cases are plotted in Fig.18. In aiding flow case, the increase in the temperature ratio parameter Cr results increase of the velocity profiles. Whereas in the opposing flow case opposite effect is found in the velocity profiles.

The effects of temperature ratio parameter Cr on temperature distributions are plotted in Fig. 19 and Fig. 20 for aiding and opposing flow cases respectively. Same effect is found in both cases. The increase in temperature ratio parameter Cr results decrease of the temperature distributions in both cases.

The effect of melting strength and thermal dispersion on heat transfer rate is shown in Fig. 21 for both aiding and opposing flows in terms of Nusselt number defined in eq (13). It is observed that Nusselt number decreases significantly with the increase of

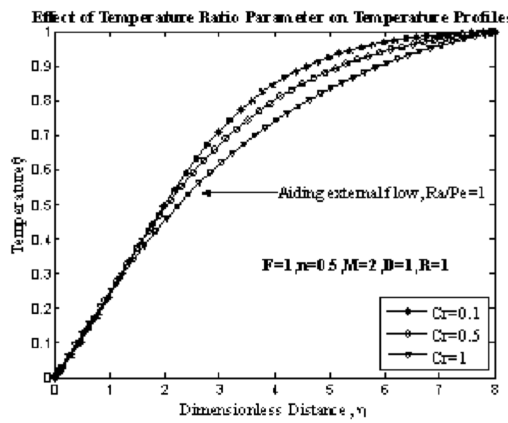

Fig. 19

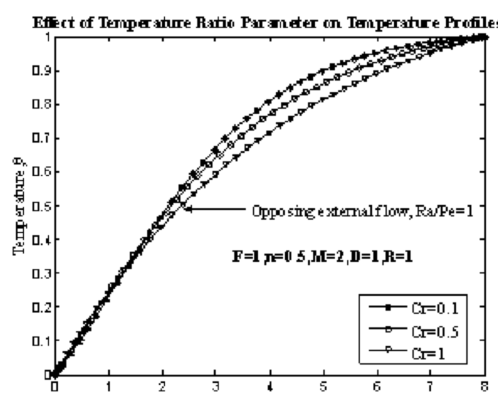

Fig. 20

melting strength M and increases with increase in thermal dispersion parameter D for both aiding and opposing flows.

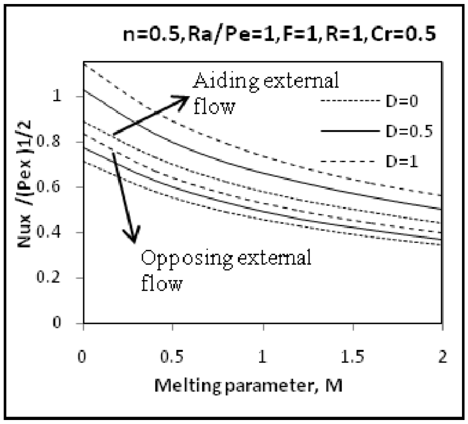

Fig. 21

The variation of Nusselt number with melting parameter for different values of non-Darcy parameter F is shown in Fig. 22 for both aiding and opposing flows. In aiding flow case the Nusselt number decreases as the non-Darcy parameter F increases. Whereas in the opposing flow case the effect is found opposite.

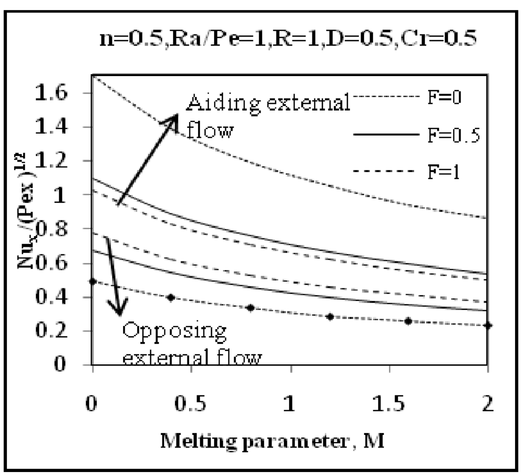

Fig. 22

The variation of Nusselt number with melting parameter M for different values of radiation parameter R is shown in Fig. 23 for both aiding and opposing flows. It is observed that in both cases the Nusselt number increases as the radiation parameter R increases.

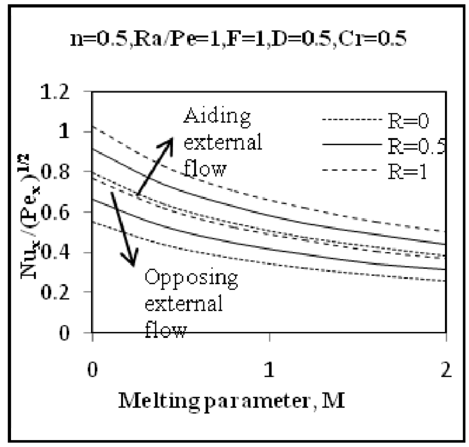

Fig. 23

The variation of Nusselt number with melting parameter M for different values of temperature ratio parameter Cr are shown is Fig. 24 and Fig. 25 for both aiding and opposing flows respectively. It is observed that the Nusselt number increases as the temperature ratio parameter increases in both flow conditions.

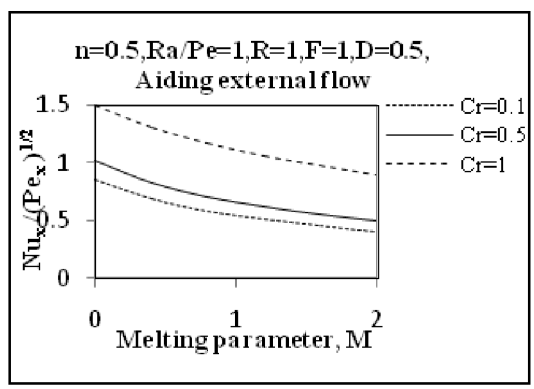

Fig. 24

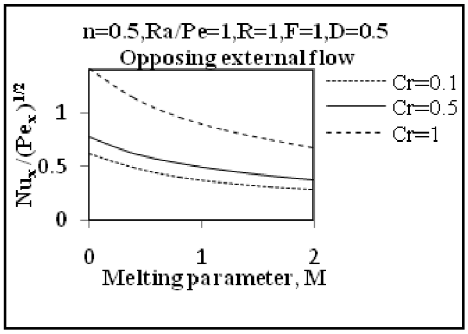

Fig. 25

We compared our results with those of earlier published works of Chemkha *et al*.[7] and Cheng and Lin[11] for special cases of the problem under consideration. Tables 1-3 show that our numerical values are in good agreement with the compared results.

Table 1. Comparison of $f^1(0)$ with values obtained by Chemkha *et al*.[7] and Cheng and Lin[11] for Newtonian fluid (n=1.0) with an aiding external flow.

M	Ra/Pe	Chemkha *et.al.* [7]	Cheng and Lin [11]	Present
2.0	0.0	1.000	1.000	1.000
	1.4	2.400	2.400	2.400
	3.0	4.000	4.000	4.000
	8.0	9.008	9.000	9.000
	10.0	11.00	11.00	11.00

Table 2. Comparison of $\theta^1(0)$ with values obtained by Chemkha *et al*.[7] and Cheng and Lin[11] for Newtonian fluid (n=1.0) with an aiding external flow.

M	Ra/Pe	Chemkha *et.al.* [7]	Cheng and Lin [11]	Present
2.0	0.0	0.27060	0.27060	0.27056
	1.4	0.38020	0.38010	0.38003
	3.0	0.47470	0.47450	0.47449
	8.0	0.69050	0.69020	0.69009
	10.0	0.75980	0.75940	0.75929

Table 3. Comparison of $\theta^1(0)$ with values obtained by Chemkha *et al*.[7] and Cheng and Lin[11] for Newtonian fluid (n=1.0) with an opposing external flow.

M	Ra/Pe	Chemkha *et.al.* [7]	Cheng and Lin [11]	Present
0.0	0.2	0.52720	0.52700	0.52694
	0.4	0.48670	0.48660	0.48656
	0.6	0.44210	0.44210	0.44204
	0.8	0.39170	0.39170	0.39169
	1.0	0.33200	0.33210	0.33205

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