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A study of oscillatory flow of blood through porous medium in a stenosed artery in the presence of magnetic field

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Abstract

The purpose of this paper is to study the effect of magnetic field on oscillatory flow of blood through porous medium in a rigid tube with a mild stenosis. Here we assumed that the blood behaves as a Newtonian fluid and the maximum height of the roughness is very small compared with the radius of the unconstricted tube. The expression are given for the instantaneous flow rate, resistive impedance and wall shear stress.

Key words: Oscillatory flow, Blood, Stenosed artery, Magnetic field and Porous medium.

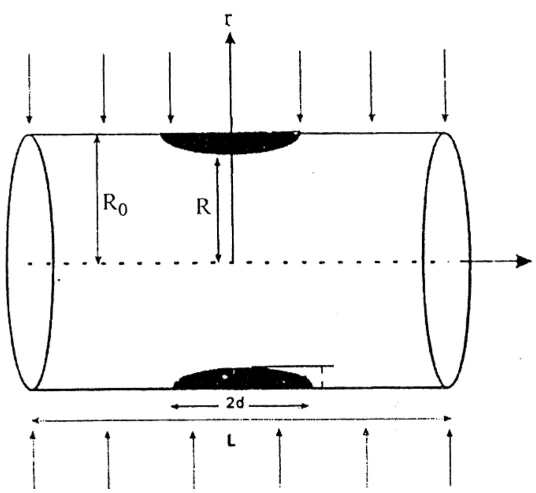
Introduction

The flow of blood through on artery depends upon the pumping action of the heart under normal conditions. The pumping mechanism of the heart gives arise to a pressure gradient which produces an oscillatory flow in the blood vessel. Womersley[13] worked on the oscillatory motion of a viscous fluid in the rigid tube under a simple harmonic pressure gradient; Daly[2] studied the pulsatile flow through canine femoral arteries with lumen constrictions; Newman *et. al*.[9] worked on the oscillatory flow numerically in a rigid tube with stenosis; Halder[3] has studied the oscillatory flow of blood in a stenosed artery; Bhattacharya[1] have also studied a simple blood flow problem in the presence of magnetic field; Mazumdar *et. al.*[8] have studied some effects of magnetic field on a Newtonian fluid through a circular tube; Tiwari[12] investigated the effect of magnetic field on a simple in an arterial region; Kumar[6] discussed the oscillatory MHD flow of blood in a stenosed artery; Quadrio and Sibilla[10] have study a numerical simulation of turbulent flow in a pipe oscillating around its axis; Rathod *et. al.*[11] have discussed the steady blood flow with periodic body acceleration and magnetic field through an exponentially diversing vessel; Liang *et. al.*[7] have studied the oscillating motions of slug flow in capillary tubes; Kumar and Singh[4] have discussed the oscillatory flow of blood in a stenosed artery in the presence of magnetic field and Mishra *et. al.*[5] have studied the oscillatory flow of blood through porous medium in a stenosed artery.

The present paper deals with the problem of oscillatory flow of blood through porous medium in a rigid tube with mild stenosis in the presence of magnetic field under the simple harmonic pressure gradient.

Mathematical Model

Consider an oscillatory but laminar flow of blood through an artery with mild stenosis which is assumed to be Newtonian. The artery is of constant radius preceding and following the stenosis. The viscosity and density of the fluid are assumed to be constant. It is also assumed that the constriction develops symmetrically due to some abnormal growth in the lumen of the artery. The idealized geometry of stenosis is given by



GEOMETRY OF THE STENOSIS

$$\frac{R(x)}{R_0} = 1 - \frac{\varepsilon}{2R_0}\left(1 + \cos\frac{\pi x}{d}\right) \tag{1}$$

Governing equations of flow in the tube are:
Equation of continuity

$$\frac{\partial u}{\partial x} = 0 \tag{2}$$

Equations of motion

$$0 = -\frac{\partial p}{\partial r} \tag{3}$$

and

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) - \left(\sigma B_0^2 + \frac{\mu}{K}\right)u \tag{4}$$

where $R(x) =$ The radius of the artery in the stenotic region

$B_0 =$ Electromagnetic induction

$\sigma =$ Conductivity of fluid

$R_0 =$ Radius of normal artery

$2d =$ Length of stenosis

$K =$ Permeability of porous medium

$\varepsilon =$ Maximum height of stenosis

$\mu =$ Viscosity

$p =$ Fluid pressure

$u =$ Velocity in axial direction

$\rho =$ Density of the fluid

Suppose that $\dfrac{\varepsilon}{R_0} << 1$

Subject to boundary conditions

$u = 0 \;\; on \;\; r = R$, no slip at the wall (5)

$\dfrac{\partial u}{\partial r} = 0 \;\; on \; r = 0$, symmetry about the axes (6)

Solution of the Problem:

The simple solution of the oscillatory motion of a viscous fluid will be obtained in this section under a pressure gradient which varies with time. A transformation defined by $y = \frac{r}{R_0}$ is introduced in equation (4) and boundary conditions (5) and (6) we get

$$\frac{\partial^2 u}{\partial y^2} + \frac{1}{y}\frac{\partial u}{\partial y} - \frac{\rho R_0^2}{\mu}\frac{\partial u}{\partial t} = \frac{R_0^2}{\mu}\frac{\partial p}{\partial x} + R_0^2\left(\frac{\sigma B_0^2}{\mu} + \frac{1}{K}\right)u \tag{7}$$

$$u = 0 \quad on \quad y = \frac{R}{R_0} \tag{8}$$

$$\frac{\partial u}{\partial y} = 0 \;\; on \;\; y = 0 \tag{9}$$

Let the solution for u and p be set in the forms

$$u(y,t) = \bar{u}(y)e^{i\omega t} \tag{10}$$

and
$$-\frac{\partial p}{\partial x} = Pe^{i\omega t} \tag{11}$$

Substituting equations (10) and (11) into equation (7), we get

$$\frac{d^2\bar{u}}{dy^2} + \frac{1}{y}\frac{d\bar{u}}{dy} - k^2\bar{u} = -\frac{R_0^2 P}{\mu} \tag{12}$$

where $k^2 = i\beta^2$

and
$$\beta^2 = \frac{R_0^2}{\mu}\left\{\rho\omega - i\left(\sigma B_0^2 + \frac{\mu}{K}\right)\right\} \tag{13}$$

The solution of the equation (12) subject to boundary conditions (8) and (9) is

$$\bar{u}(y) = \frac{PR_0^2}{i\mu\beta^2}\left\{1 - \frac{J_0\left(i^{3/2}\beta\, y\right)}{J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)}\right\} \tag{14}$$

where J_0 is the Bessel function of order zero with complex argument, then the resulting expression for the axial velocity in the tube is given by

$$u(r,t) = \frac{PR_0^2}{i\mu\beta^2}\left\{1 - \frac{J_0\left(i^{3/2}\beta\frac{r}{R_0}\right)}{J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)}\right\}e^{i\omega t} \tag{15}$$

The volumetric flow rate, Q, is given by

$$Q = 2\pi\int_0^R ur\,dr \tag{16}$$

which gives, on integrating, at once

$$Q = \frac{\pi R_0^4 P}{i\mu\beta^2}\left(\frac{R}{R_0}\right)\left\{\frac{R}{R_0} - \frac{2J_1\left(i^{3/2}\beta\frac{R}{R_0}\right)}{i^{3/2}\beta J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)}\right\}e^{i\omega t} \tag{17}$$

The shear stress at the wall $r = R$ is defined by

$$\tau_R = \mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

which gives

$$\tau_R = \frac{R_0 Pe^{i\omega t}}{\beta}i^{3/2}\frac{J_1\left(i^{3/2}\beta\frac{R}{R_0}\right)}{J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)} \tag{18}$$

now
$$\frac{\tau_R}{Q}=-i\frac{\mu\beta^2}{\pi R_0^3}\frac{J_1\left(i^{3/2}\beta\frac{R}{R_0}\right)}{i^{3/2}\beta\left(\frac{R}{R_0}\right)^2 J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)-2\left(\frac{R}{R_0}\right)J_1\left(i^{3/2}\beta\frac{R}{R_0}\right)} \tag{19}$$

If τ_R is normalized with the steady flow solution given by

$$\tau_N=\frac{4\mu Q_0}{\pi R_0^3}$$

where Q_0 is the steady-flow volume rate.

The resistive impedance to the flow is defined by

$$z=-\frac{\partial p/\partial x}{Q}$$

$$\therefore \quad z=\frac{i\mu\beta^2}{\pi R_0^4}\frac{\left(\frac{R}{R_0}\right)}{\left\{\frac{R}{R_0}-\frac{2J_1\left(i^{3/2}\beta\frac{R}{R_0}\right)}{i^{3/2}\beta J_0\left(i^{3/2}\beta\frac{R}{R_0}\right)}\right\}} \tag{20}$$

Deduction :

1. If porous medium is withdrawn *i.e.* $K=\infty$ then all the results are agree with Kumar and Singh (2008).
2. If Magnetic field is withdrawn *i.e.* $\beta=0$ then the all the expressions are agree with Mishra *et.al.* (2012).

Discussion:

From figure 1 it is clear that instantaneous flow rate is going to decrease as soon as frequency parameter β is going to increase. We have also calculated instantaneous flow rate for the different values of stenosis height. We observe that the instantaneous flow rate is going to decrease as stenosis height is going to increase and it is also very much clear from the figure that instantaneous flow rate becomes steady when frequency parameter β is large.

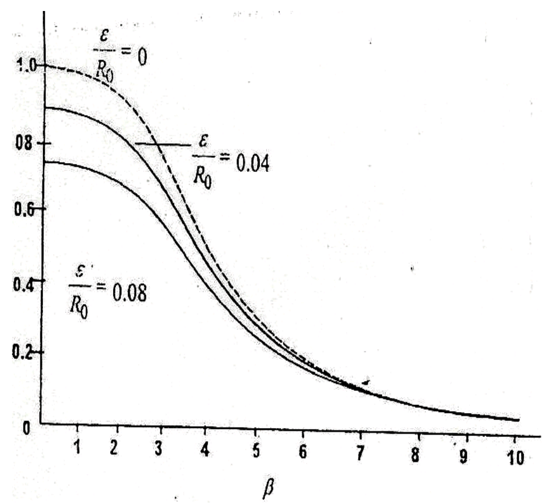


Fig. 1. Variation of instantaneous flow rate with frequency for different value of stenosis height through porous medium in the presence of uniform magnetic field.

For $\beta = 0$ it has a maximum flow rate and there is no major change when β increase upto a value less than unity.

Figure 2 shows that the impedance is zero for $\beta = 0$. The impedance is going to increase as β is going to increase. There is no major change in the impedance when β is lying between 0 to 1. It is also observed that impedance is increased with the increase of stenosis height.

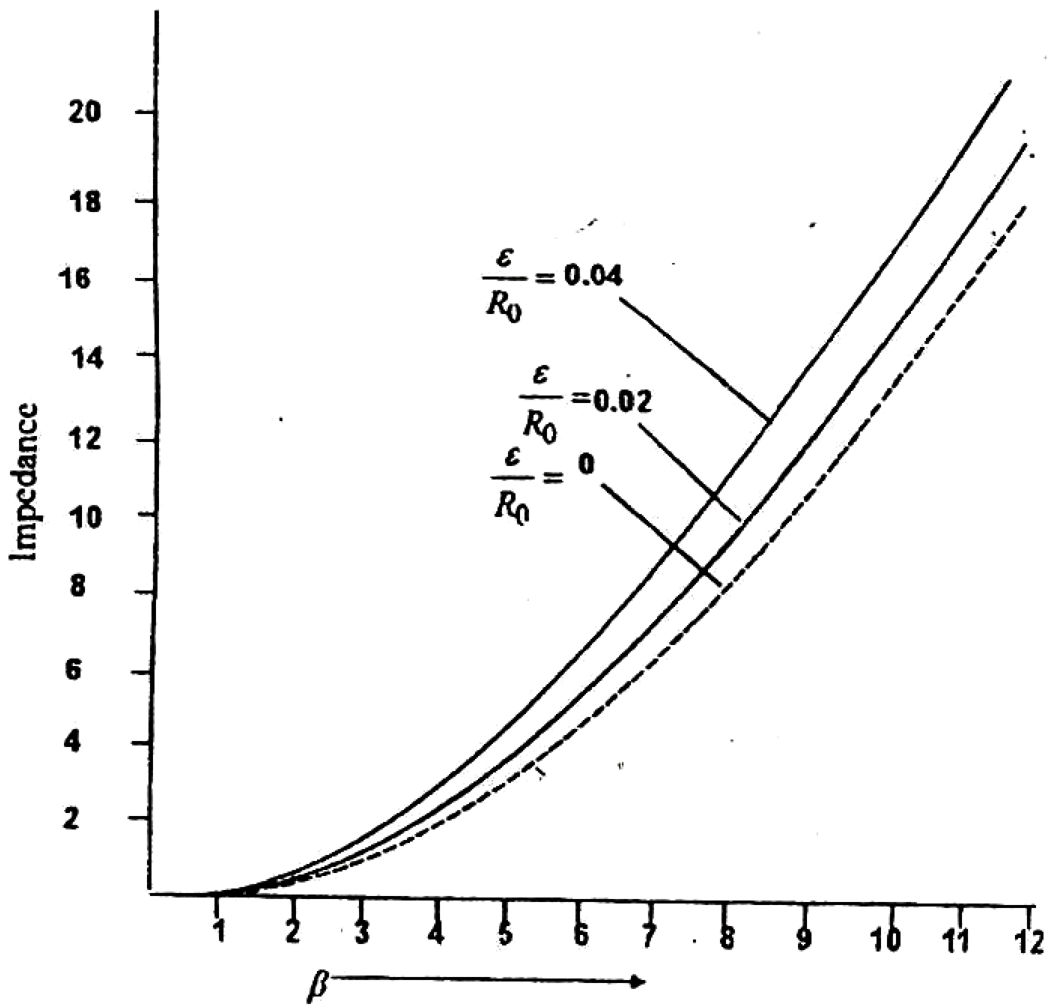


Fig. 2. Variation of normalize impedance with frequency for different values of stenosis height through porous medium in the presence of uniform magnetic field.

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